talk by ralph abraham, intl synergy inst, santa monica church, july 1992

Sacred Mathematics and the Chaos Revolution

In the neo-Platonic cosmology, which was revived in the renaissance, mathematics had a special place. It lived kind of halfway between heaven and earth. Since the Enlightenment, mathematics has lost its sacred place, but it still offers what it always had to offer to those who want to make use of it. It's sort of a mid-station between heaven and hell, a mezzanine floor in the spiritual hotel. If we are partially involved in archaic revival, if we are seeking the lost values of partnership society or the pagan world, then the resurrection of the sacred role of mathematics, along with other sacred arts, will be an essential step toward creating a future.

It seems odd and wonderful to find an audience coming out on a Friday night for a talk on mathematics. We might ask, why would we be interested in mathematics, in dynamical systems theory, in chaos theory, in mathematical models, in computer simulation, or in computer graphic visualization? Is there, indeed can there be, a sacred role or a secular role for this knowledge in view of the evolutionary challenge at this time, and of our problem to create the future, or even to have a future? So before going on, I want to try to give a fraction of my own answer to the question of the utility of mathematics for sacred and secular purposes and the creation of the future, as well as my main motivation in doing this in my life instead of just going to the beach.

Over the course of pre-history, there are different kinds of mathematics in the sacred tradition, and they are still growing; lately they have grown new branches devoted to the understanding of the space-time pattern. The most recent developments have unrolled in the context of the computer revolution. Therefore it is no accident that we have a new mathematics of the space-time pattern at the same time that we have the means to see it. This development couldn't have taken place until computer graphics became available. Which of these might apply is a question for historians of mathematics to unravel in the future. We don't know. All we know is that it has happened primarily with people who had different kinds of advanced computers in their office, under their desk, in the basement, with some kind of access to new technology, with the capability to make mathematics visible.

Now it could be that when the computer revolution is evaluated many years hence, it will be valued primarily for its role in creating virtual reality for science fiction films. Or it may be remembered as the day in the history of the species when everyone got to see mathematical reality, which had previously been visible only to a few specially-trained people who could see it only as long as they maintained exercises that went on all day and never had a single moment of ordinary life, sort of like monks. That's what the situation of creative mathematicians was like in the past. For us it will be enough to understand computer graphics as a telescope into a mathematical landscape previously hidden and occupying some intermediate space between heaven and earth. Through this looking glass, we can see most mathematical

objects previously known, from Pythagoras to today, and we can also see new ones. The new ones had not been seen before computer graphics.

Part of the magic of the chaos revolution is that we've learned ways to employ mathematical theory and high-speed computers to see and understand a level of complexity that was previously beyond our grasp. If we think of complexity in some intuitive way as an attribute of a dynamical system, a relationship, a society or whatever, and if we could make an easy intuitive judgment that some of these are more complex and others are less complex, then we'd have an entire universe of perceptive reality. We have a kind of radius function that allows us to imagine the more complex systems being farther away and the simpler systems being closer. In the universe of ordinary reality, some of us can see a certain distance. Others can see a little farther in one angle, like, say, the musical realm, and not so far in another way, where the graphic artist is trained. Overall, certain animals, like migrating birds for example, might be able to see a whole lot farther than we can. But for every individual, there is a kind of limit, a complexity horizon. In this perspective, chaos theory and the computer have given us the chaoscope, a kind of telescope, with which we can see farther -- our complexity horizon is advanced. The whole universe of complex space-time patterns that we can grok is now increased.

Whether it's increased enough so that we can now understand a relationship, an ecological system, the planetary biosphere, the global economy, the emerging community of nations, I don't know. But it's a step in that direction and as such, can be used by us to train ourselves and our children to have an intuition and to grok systems of greater complexity, enhancing at least a little bit our capability to have a sensitive and successful co-existence with our environment on the dying planet. Well, that's a little fantasy. I can't convince you of this, but anyway you have an idea now, I think, why I have a passionate obsession with this work.

In the new mathematics we have a tremendous closet-full of models of complex space-time behaviors. If we wanted to treat them as atoms and put them together into a molecule, we'd have to take a whole bunch of them and connect them up. Today we have supercomputer technology which is ideal for this. It is called *The Massively Parallel Processor*. When we did this work three years ago there was only one in the world, and it was at NASA Goddard Space Flight Center. We went there, and we coupled together 16,000 dynamical systems —- the simplest dynamical system which exhibits chaotic behavior. We coupled them in a two-dimensional array, like a television screen where one whole computer is devoted to every point on the screen. It's like putting a bunch of chaotic tops in a bag, pouring them out on the tabletop, allowing each one to contact the four nearest neighbors, and then starting it off with some pattern. Some of the tops would spindle up and others would spindle down, and this mess is a field of chaos if you just let it go and see what results.

This was envisioned as a mathematical model for morphogenesis, which is a general idea on the emergence of form in whatever realm —- a physical system like the solar system, galaxies, or a biological system like a chicken egg, a single cell turning into a chicken, or a social system, where cities, farms and fields emerge in a desert or in the Amazon. The idea was just pure math exploration, and this is what resulted. (Video being shown) This is a map of a two-dimensional space that we're able to navigate. The overall supercomputer

experiment has two parameters which we control, and everything else is intrinsic to the evolving system. We envision this two-dimensional space as the surface of the ocean. Choosing one position on this space, we could let down a bathyscaphe with a television camera, and that would unroll a dynamical system in which each picture leads to another one according to a fixed dynamical rule, depending on the position of our research vessel on the ocean.

This is a so-called scan, and that means that we descended to a depth of 1,000 feet or 1,000 iterations and took a single snapshot. Later we assembled these snapshots into a slide show that demonstrates what results after a long dive, starting from different points in the ocean. So this is not actually a dynamical system but what we called a scan, just an encyclopedia of results -- the slide show flashed at the rate of 10 slides per second. In different regions of the ocean we dove and discovered different personalities of pattern, different characteristic patterns, with more or less spacial organization or spacial chaos. So here is a region with more spacial organization. We liked this area so we gave it a name.

There's a lot of symmetry because we started with a symmetric initial patten, which you don't see because these are each snapshots after 1,000 seconds. When we started with a random initial pattern we got a much more chaotic result. We started out with a regular pattern in the Sea of Chaos. We chose a fairly standard color palette. The supercomputer is computing a number at each location, and we just chose red for one and blue for zero. So as you'll see later in the dynamic sequences, these patterns result just after a few steps in the iteration of the dynamical system —- let us say successive generations of a society. These highly-ordered patterns result in a short time from a chaotic field.

We have new mathematical models for chaos, and I describe them right now in this situation, but they're pretty simple. It's a very easy program to run on a supercomputer. So what is happening? This, by the way, is a dive, so here we see the initial pattern and more or less the step-by-step emergence of a stable form in the field of chaos. So instead of getting more and more chaotic, it gets more and more ordered. That's not exactly a mathematical law of nature, but this is what we've observed in all the systems we studied. Order tends to emerge in these systems. At each point in the screen the computer is computing a number -- between zero and twenty, let's say —- and each time it computes a new number it uses the last number as a starting point. So it's a kind of self-referential system or iteration. And these city plans emerge.

If there was no communication between the computer at one location and the four neighbors, these highly-ordered forms would never emerge, and of the two parameters that we change at the beginning of each experiment, this one is a new dive with different values of the parameters. The main parameters are: the degree of chaos at each point (that's one number common throughout the two-dimensional array of systems) is one parameter, and the other is the sensitivity or the strength of the coupling between each individual and the four neighbors. So when there's no coupling at all, you see no pattern emerge, and as soon as there's a very small coupling, that's enough for a cooperative pattern to self-organize out of the chaotic field.

When applied to a model of society, we have at the present time a situation where all the different institutions of society seem progressively less related to all the other institutions -- where our federal government has very little relationship to the ideas of the population and where the different world governments are increasingly losing their control over their component parts. The strength of coupling in a hierarchical, patriarchal order throughout the planet on a social scale is decreasing. If this coupling should reach zero, we would indeed be in the chaotic state that many people are afraid of, where we couldn't actually trust any structures that we're familiar with.

As soon as there is just a little bit of coupling, however, then according to these experiments we can be confident that, as in the metamorphosis of a butterfly from a caterpillar in the cocoon, a new order will emerge from a field of chaos. All we need is a little bit of coupling. That's kind of the implication of this system —- it's a very abstract model -- for a society which has a large number of independent agents, say on the order of 16,000. It's a dynamical system, and there are no random elements. So in spite of the fact that there's a chaos generator at each point, every run produces exactly the same result.

Self-organization is an old and traditional field in science, especially in the biological sciences. Processes like neurogenesis and the organization of the physical skeleton, the spine and so on, are outstanding problems in biology that have not yet been solved. In social systems, the self-organization process is even less understood because we have no concept of specific physical processes that would organize a social system. In the biological system we have weak electrical fields, we have neurotransmitters and so on, but in social systems we don't know what the forces would be, although there are some clues in the work of people like Byon or Kurt Levine. In these totally abstract mathematical systems we see, without knowing if it is a kind of a conceptual or cognitive understanding of the forces, how patterns emerge simply on the basis of mathematical rule.
[End of that video]

Under the name morphogenesis or pattern-formation there has been an outstanding (?) that emerged first in embryology and later in population dynamics. Mathematical models had been developed for certain pattern formation processes, but such patterns as the spiral galaxies in the universe, the solar system, certainly the embryo of a chicken —- these pattern formation processes have no model and no physical theory. The biologists do not have a theory. There are certain speculations. Rachevsky, for example, in the ____, proposed that weak electrical fields would have enough structure to allow the development of a chicken cell into a chicken. The molecular biologists, the people who believe that every aspect of the individual biological specimen is controlled by his DNA structure, believe that the DNA actually directs the process by unknown physical means, but definitely controls every step of the embryogenesis process. But how? We don't know. Some combination of weak electrical fields, biochemical transmitters, construction of proteins that then interact; it's an outstanding process.

What we see now is that mathematical objects alone have patterned formation rules. Given certain kinds of interconnection, a certain kind of pattern development will necessarily result. Even if we don't know how the DNA, for

example, could control the interconnection of all these elements, mathematics will show that with a given structure on, let's say, a plane of guidance fields, a given form or space-time pattern will emerge after an iterative process in which each form develops from a previous form. Let me read you a sentence from Will McGwinney's talk at a meeting we had three years ago. He is speaking about two different models for the emergent social order. One is called The Holism Model, and the other, The Arabesque:

The Arabesque paradigm is an evolving culture, creating itself by unending iterations of processes that make distinctions. It creates by differentiation. Consider for example the creation of language: from the awareness of such characteristics as light, structure, weight, force, odor and so on, we have gone on through metaphoric projection to produce an endless variety of symbols that make up each language.

Here is an attempt to describe the microscopic, internal, mechanistic steps that would go on in the emergence of form, in morphogenetic process, in society, where the process, the time evolution of the society under consideration is discrete steps, say generations. The social organization of a given generation is created from the social organization of the previous generation, without the necessity of knowing its entire history, according to some unknown rule of social evolution, which at least is described metaphorically in the context of this metaphorical example of language. Later, McGwinney says:

An arabesque is a literary simile of a fractal, a mathematical form which not only models many of the qualities of our present society, but also many qualities to be found in the coming century. Like the arabesque, the fractal is a figure generated by the repeated application of a rule or a variety of rules. Such figures may be of incredible richness, which even if unendingly elaborated, can never be fully bounded and never fully occupy the spaces they appear to span.

There's a suggestion here, worked out in some detail, for particularly interesting aspects for the future, such as the relationship between good and evil elements of a society, where all the metaphors used in the description somehow are those that are studied in the abstract in these mathematical models. In these models that I show and the coupling rule that I explain, there is an equal strength of the coupling at every point between the system and the four nearest neighbors.

Have we experimented with couplings to more distant neighbors and also experimented with changing the strength of the coupling from point to point? Well, that is the idea of a neural net, which is a structure just like these but more complicated. And neural nets have so many parameters that the spaces of models can't be studied by exploration. So instead, automatic programmers have been developed, which compare the behavior of a system like this to a desired result, and automatically change all these zillions of parameters to try to adjust the behavior of the dynamical model to match data from the laboratory. That's an elaboration of this kind of two-dimensional world into a multi-dimensional world too complicated to immediately grok.

What we're trying to do now, on the frontier of mathematics, is to find a convenient compromise between the complexity of the actual universe and the

maximum complexity we can understand. So we try to study models that are simple enough to understand and yet complex enough to have interesting behaviors that are suggestive of space-time patterns observed in the wild, in our lives. Human beings are really simple, and our understanding is very limited. The world that we live in is not the most complex thing possible, but it is already more complex than the most complex thing imaginable. And that’s probably good.

I don’t believe that we have a great chance of really grokking the universe around us, ever. I think we are too stupid to survive on this planet. If we can enhance our intelligence just a little bit, either with machines, with meditation, with twelve steps, with the new curriculum in K through 12, by whatever means, then a very small increase in our intelligence and the understanding of space-time patterns could result in a very big gain in terms of our survival. Is there a relationship to entrainment? There is, but the relationship is more explicit in terms of another model which we studied more recently, so I’d like to show you that.

Frequency and phase entrainment are ideas from mechanical engineering at the turn of the century. Clocks don’t keep very good time. If you compare two pendulum clocks, they are going to be fast or slow, maybe they’re both fast or slow, but not by the same amount. So these clocks will not keep the same time. But it was discovered that if you put them on a table, they’ll keep the same time, and that’s called "frequency entrainment". The two oscillators, the pendulums that, separated, would oscillate at different rates with a very weak connection between them by whatever mechanical method –- it’s carried through a table top from one pendulum to another –- would kind of synchronize their frequencies so they kept the same time, whether fast or slow. They would gain time at exactly the same rate. Now, the frequency entrainment of pendula could happen different ways. For example you could have this pendulum and that pendulum going so they keep exactly the same time but there’s no simple relationship between their phases. However, that’s not what’s observed, because they get not only frequency entrainment, but sooner or later they also get phase entrainment, so they go back and forth simultaneously.

These are two different phenomena under the name "entrainment" that apply to mechanical oscillators, electrical oscillators, biological oscillators, the synchronicity of a menstrual period in a group of women, and so on. So these concepts of frequency and phase entrainment apply to continuous dynamical systems that are oscillators. We did another experiment where we took 16,000 of those,[WHICH?] and connected them up the same way. This is the particular model for a continuous oscillation called the [bruscillator?], which may apply more or less to chemical reactions like those involved in the ozone hole. It has been suggested as a model for biological morphogenesis and was also applied to social systems by Prigogine. That was his main idea, for which he got the Nobel Prize.

I’m going to describe two projects. The first one is an ongoing project in which there are many different experiments to be done and only a few part-time people to do them. Since there’s no way to do them without the world’s fast supercomputer, the project goes along slowly. However, supercomputers are becoming smaller and cheaper, and if we know already what the landscape looks like and we just want to reproduce one experiment, we can do it in real time with a portable supercomputer. And therefore it’s possible to use such

systems as a medium for visual music and to perform live with a keyboard. That's the concept of the so-called MIMI, the mathematically-illuminated musical instrument. The idea is using mathematically-defined dynamical systems with supercomputer simulation and computer graphic visualization as a performance medium for the visual arts. The field of visual music has a long tradition, stretching back to prehistory, and in recent times, let's say during the 19th and the 20th century, there have been famous performers who, with a kind of apparatus including candles and clothes, have given silent performances for audiences. Since the advent of color film there have been some really beautiful works of visual music arts created by frame-by-frame animation, for example by moving dolls around on a table and photographing them, like Oscar Fishinger did.

We'll give a concert of visual music with this MIMI, thanks to the loan of a portable supercomputer from Silicon Graphics, Incorporated. It will take place on October 17th in the Cathedral of St.John the Divine in Manhattan, the largest Gothic cathedral in the world and the second most popular tourist attraction on Manhattan Island. I view this event as somehow more important than simply a kind of public masturbation of supercomputer graphics. I imagine that since these are forms that emerge from chaos and comprise the actual notes, the visual music that's played in each sequence is a kind of sacred art which leads to an understanding of the whole concept without having to read a mathematical book.

Mathematics, when made visible through computer technology, is beautiful. It always was beautiful but it wasn't visible, so visible mathematics is an important artistic event that extends the landscape of visual materials available to the artist enormously. There's a whole parallel universe, ordinary reality and the mathematical mezzanine floor on the way to the top floor. Furthermore, the fact that it's taking place in a cathedral suggests the sacred application of the reconnection of people to their spirit through the recognition of mathematical forms as perceived experiences, and the possible application to the future evolution of society and the economic and environmental systems as well as the coupling between environmental and economic systems. We don't know any other way to bring a biosphere into a church. The Cathedral of St.John the Divine intends to have a biosphere, a bioshelter they call it, a piece of Gaia, inside the church. In order to make that possible they have to re-define Gothic architecture, changing the number of floors from three floors, representing the three levels of the Pythagorian or Renaissance cosmology, to four by adding a new floor for Gaia. The invitation by the Cathedral of St.John the Divine for this particular performance of real-time, mathematically-illuminated visual music is motivated by their desire to re-create the Church, to re-sacralize and regenerate the Church through the inclusion of the Green idea. Anyway, that's one application that we're working on.

And now I'd like to go on to my second project. Could the entrainment of the frequency of vibration have anything to do with the formation of crystals? Well, yes. We saw in the first video that there were a lot of symmetries—four-fold symmetries, radial symmetries, toral symmetries. So what's the cause of this extreme symmetry? The fact is that if you start with a symmetric configuration, and the rule for changing one picture into the next one has symmetry built in, then the results will always be symmetric. That's the situation in crystal formation, where the symmetries of crystals are

classified by mathematical systems called the crystalographic groups. There is a tradition in the physical sciences where these kinds of models have been used to explain morphogenesis in solids, whether of crystalline or glassy structures.

There are examples in every sphere of morphogenesis throughout all the various sciences, and right now I'm specializing in the social sciences because, first of all, mathematical aid has not yet arrived in the social sciences, and secondly, it seems to be the area where we need the most aid, mathematical or otherwise, in order to survive our current problems.
[QUESTION ... UNINTELLIGIBLE] [SHOWING SECOND VIDEO HERE]

The oscillators are trajectories into two-dimensional space, representing the concentration of two different chemicals among three chemicals that are imagined to be in oscillation. We can't show the two different concentrations at each point of the two-dimensional physical substrate simultaneously, but we know that these oscillators are going around some kind of more or less round curve. If we take a fixed point on that curve and call it the zero phase, then we have the phases zero, 90 degrees, 180 degrees, 270, 350 degrees, phases that are kind of an angular coordinate in the two-dimensional plane of the chemical concentration. We show only that angular coordinate. That means that you see some blue here and some blue down here. All of those oscillators are exactly in phase. But the red oscillators and the blue oscillators are exactly out of phase.

The phase relationship in biological oscillators is very important. For example in the menstrual cycle you have in the middle a catastrophic event called the LH surge. A huge amount of chemical transmitter—luteinizing hormone is released sort of all at once. How can that happen? You have a whole bunch of chemical oscillators in the pituitary or somewhere, and they each have little sacs of LH which are to be released and then they're oscillators, and then they have to know to do this release at the same time. There's a very flimsy communication between them, and if they didn't release all in phase, if that region wasn't all blue, then first of all there wouldn't be an LH surge.

Now you've got another event, that is the red event of ovulation, and then these are two weeks apart. The phase relationship between them has to remain 180 degrees; otherwise, life would be impossible. In the first experiments of entrainment ever done, Christian Hoynkens [sp?], the inventor of the pendulum clock in the year 1623, was trying to get a barrel escapement mechanism to work, and he noticed this entrainment. He wrote about it in a letter to his father in Paris, and he tried an experiment. He waited until the pendula were in frequency entrainment and phase entrainment, and then he grabbed one and held it, and then he let it go. He measured the length of time it took to come back: 20 minutes. That's just two oscillators.

We have tried a lot of experiments with this system, and depending on the choice of the various other parameters, it takes longer or shorter for the system to re-establish the same picture. That is an expected result under an existing theory of dynamical systems that I haven't talked about -- the concept of attractor and basin. These dynamical systems have many different attractors, so starting with different initial conditions, that means the first pattern. Suppose we start with a blue circle in a green field, for

example. If we had started with a different one, then the pattern that comes out wouldn't look like Stonehenge at all, it'd look like Avebury. You'd get a completely different result. So where you start and where you end up is called the attractor-basin portrait of the dynamical system. It's the most important thing to know about a dynamical system if you wanted to be able to survive. The attractor in one basin might be death, and the other basin and its attractor might be life, and you'd like to know, if you were in the wrong basin, how you could get out of it into another one quickly -- but not too quickly, like leaving a building in case of an earthquake.

The question is, why did the first video always look like a kaleidoscope? The answer is, it was intentional, because it simplifies the overall pattern to the extent that it's easier to grok what's going on. We did some other experiments, where we started with a very irregular initial pattern, and the result looks like a wriggling bowl of spaghetti. It still represents the same degree of space-time organization, it's just not as easy to grok it, because, well, can you tell one bowl of wiggling spaghetti from another? It's something like counting the number of beads in a jar, but with training you could do it. With the kaleidoscopic symmetry, it's much easier for all of us to understand what's going on.

Can the concept of entrainment and chaos theory help us to understand the psychological and social chaos we are presently experiencing? There are several different levels. In fact that was kind of the idea of my talk about my personal motivation. My personal motivation in doing this, in part, is that the answer to that question is yes. Some people, observers of psychological and social chaos, have actually come to me for specific aid in a project. For example, psychotherapists have noticed that their patients give a pre-warning of a break in a kind of irregularity —- say, in lateness to appointments, or checks bouncing and things like that. So they came to us asking, "If we give you data, can you quantify how to actually predict breaks?" In principle, the answer is yes. But practically, we found it very difficult because it's hard to get the data.

Maybe psychotherapy could develop a mathematical branch that had specific models and predictions and so on. But I think that would be less important than a more general level of help, where just the basic concepts are helpful. For example, if you are brought up with the paradigm of modern scientism, then order is not allowed in experiments. In a lot of scientific labs, in fact, even periodic oscillation is not allowed. So when it appears, the data is jettisoned. But if you can accept that there are mathematical models for chaotic behavior and change your paradigm so that you allow yourself to observe chaos in your life or your practice, then you've got maybe the greatest gain that chaos theory can give you. It's just a paradigm shift that allows you to realize that there are different kinds of chaos which you can recognize, just as you recognize the faces of human friends and cat friends and so on. These are very complex pictures —- we do have a space-time pattern recognition facility, not much developed in the visual arts but very extensively developed in the field of music. And once we get the idea that chaotic patterns, noisy patterns, are scientifically and personally useful, then our complexity horizon has been extended.

Nowadays a large number of composers are using chaotic pattern generators to generate sounds, to generate music by using the chaotic pattern on every level

in the hierarchy of musical structure. In the performance at the Cathedral of St. John the Divine, each of the three of us -- one on supercomputer, one on sound and one on link -- have different modes that we are going to use. One of them is that the person playing picture will be given total control of the sound, so that from the picture —- for example, by grabbing a stick and scratching it across the picture, each color giving off a different sound —- the MIMI messages will be sent to the synthesizers and you'll see the picture in the way that the operator has chosen by scratching. Or you could take an instrument like a trombone or a clarinet and just park it at one place in the picture, let the colors in that pixel of the picture choose the tone or the timbre and so on. Another mode is to allow the musician who is a cellist with an electronic cello to control all the parameters of the visual display from the cello. And then there's another mode where all three people will play independently as humans, meaning that the linking between the three elements would be entirely the traditional one of jazz musicians or Indian musicians doing improvisation together.

My other area of activity at the moment is fired, much to my surprise, by a visit to an international conference of economists. I was paid a lot to go there and give a tutorial on dynamics, otherwise I would never have gone because I just hate economics. Everybody has their plus and minus fields, and mine is economics. But I went, and was amazed to find that these people were very professionally involved on the research frontier of pure mathematics. Coming from an economic, mathematical modeling orientation, they had discovered a frontier of mathematics that mathematicians didn't know about, or had somehow ignored. Eventually, in order to relate to them, I had to confront my prejudice against economics, and one way I did that was by looking in the dictionary.

In Webster's deluxe unabridged, it says, "Economics, from Greek 'economicos'. It's the management of a household or state, from 'oikos', house, and 'nomos', management." So it's not just the money. It's also the buying of groceries and doing the new roof and painting walls and stuff like that. Meaning 1) "The management of the income of a household's private business. 2) The production, distribution and consumption of wealth. 3) Economics . . . " and so on. That's Webster. Partridge's Origins says, "Economic, whence economical, economist, economize, economy, ecumenical, whence diocese, parish, parishioner, parochial, and parochialism." All of these words, from Greek "oikos", a dwelling place or house. "Oikos" occurs in "oikinomos" and "oikinomia". There's a whole textbook on "oikinomia", which is a fundamental word in Christian theology. This book is called *God for Us*: *The Trinity in Christian Life*.

Trinity is an idea in early Christian theology which somehow got lost. It wasn't fully established until the Council of Byzantium in 389 A.D., and then many years later, it was lost. Now people are trying to revise it; it's an intrusion into early Christianity from the patriarchal society of the distant past, with the goddess Trivea. In her book, Lechudna has a chapter called "The Meaning of Economia." The word economia is from economeo. Originally it had the purely secular meaning of administering and managing goods or a household, or overseeing an office according to some plan or design. Oikinomia is also used to mean "the plan of salvation," or how God administers God's plan. In Epheseus, economy refers to the mystery of God's benevolent will or plan of salvation, hidden for all eternity but made manifest in

Christ. In general then, "economy" refers to the plan made known in the coming of Christ. "Economy" is the actualization in time and history of the eternal plan or of redemption, the providential ordering of all things. It suggests morphogenesis, it suggests the salvation from our current environmental crisis.

The meaning of the word has evolved from Greek antiquity until now in this context, and only in the past century did it get to mean what we mean by economy. According to Lechudna, "the term 'economia' was used broadly in the early Church. A few basic meanings can be discerned. First, 'economia' means God's providential plan, or ordering of the cosmos." So even the creation of the universe in Genesis I and II is a matter of economy. In fact, all of our experiments are economical. Second, by the end of the third century, "economia" is more narrowly understood as a synonym for "incarnation," and third, "economia" means the proportion and the coordination of constituent elements, as in the distribution or economizing of Godhead among divine persons. In other words, the three elements of the Christian trinity economize equally the wealth of the original Godhead.

So after reading up on these meanings, I accepted the invitation to lecture at the conference on economics. Their program was to study the dynamics of a new frontier of mathematics, called "endomorphisms". It produces pictures which are particularly understandable. A new technology has been discovered to understand them, called the Method of Critical Lines. One possibility lies in the future development of this subject of very complex systems. For example, we have a model for the economy of a nation; typically it would have 1,000 institutions. So you already need 1,000 computers to model the economy of one nation. Now we have 250 more or less nations, so altogether we need 250,000 computers. Well, is there any hope to understand such a complex dynamical system? There is hope, because this particular method allows us to project the entire enormous, humongous dynamic down into two-dimensional space and actually grok what's going on.

RALPH - I DIDN'T TOUCH THE FOLLOWING BECAUSE IT GOES WITH A SLIDE SHOW.
So let's look at these slides. We just constructed a new telescope for endos, which is a computer graphic program. Here is my student Ron Record, who wrote the program, working at a typical frame on a deck station, a relatively inexpensive engineering workstation. Next. The opening screen of the program. Next. A menu where we have a lot of different maps that we're doing experiments with and different possibilities for studying each one, some of which we'll see in a minute. Next. Here we see—the menu allows us to generate a large number of different windows seeing the same dynamical system from different angles, as it were. Next. The usual thing we use is called the Method of Critical Curves. It's a simple computer algorithm that generates a kind of envelope in which the chaotic attractor, or the observed trajectory or observed behavior, will be contained. Next. Here are some critical curves for one map that we're studying in great detail, called the Twisted Double Logistic. Next. The little white areas of the previous one are expanded by application of a map for successive generations and eventually creates some kind of envelope like this, which will contain the chaotic behavior. Next. That was a critical curve window; this is the histogram window. It shows the trajectory when we have iterated the map hundreds of thousands of times, and on each pixel of the screen, count off how many points are there and then use blue for a sparse zone and red for a very popular zone,

in the two-dimensions. So it's a histogram, colored trajectory of a chaotic attractor that lives entirely within the curves shown in earlier windows called the critical curves. Next. They make very beautiful pictures. We have, in all of the maps that we study, some control parameter that can be changed—for example, by dragging an elevator bar on the screen—and we get many different patterns of chaotic behavior. Next. Another one, again. Next. We want then to study the universe of all possibility and give some indication of what would be found, and this is one experimental method called the Leoffenoff Exponent [sp?]. We have a two-dimensional space of possibilities here, where each point, as in the first video, controls the entire dynamic. And we ask whether the result is chaotic or not, and if not, whether it is periodic and what is the period and so on. And color on the screen—red if we find a chaotic attractor. So the computer goes off and computes for a long time, a minute or two, and reports back with a summary of all the behavior that will be observed in this entire family of dynamical models. Next. Then we have an even better picture of the behavior, again in the same two-dimensional space of all possibility, in which a kind of snapshot of the actual attractor—not just that it's chaotic or not—is given for all the parameters, called the Bifurcation Diagram. And I don't know why they have these shapes—we just discovered them. Next. This is a similar picture for a different family. Next. And that's the end. Another Bifurcation Diagram for a different family. So this is the current frontier on a new chaoscope, a way of trying to image, in complexity that we can recognize using our human pattern-recognition capability, recognize and grok complexity in hugely complex systems. Okay, the lights. Oh well I have a video of this—maybe we should show the video and we don't have to turn the lights off. So, if you can start the video without turning the lights off, please do. Is it possible? So this is one map, my favorite one at the moment, the Twisted Double Logistic, from which we've developed a particularly complete picture of this behavior. And reduced it to video with a special technique of scientific visualization that has been developed here at NASA. [Videotape plays] For the first time we show the equations. X and y are just real numbers and, with just a little arithmetic from an old x and y, you get a new one. And each is visualized as a point in the plane. So the dynamic makes, generates a sequence of points in the plane that move around and form those particularly beautiful patterns. And here they're all assembled in full complexity in three-dimensional space, where the control parameter C is now going vertical. So, when C is zero that's at the top, you have very simple behavior with only three points, and after passing a certain critical value, suddenly the behavior becomes chaotic, but only in fairly small regions, and then the chaotic region grows and grows as we get lower and lower in the picture. And here also, the color indicates the frequency with which a certain region is visited by the dynamical system, with blue being more frequent and green medium, and yellow less. This is the enlargement of one particularly interesting region, where down at the bottom we have what is called a periodic window. And the method of critical curves that I mentioned before—that's the main actor in our interactive program endo—the critical curves allow you to predict when such bifurcations happen that simple behavior suddenly explodes into complex. And this is the kind of thing that might be useful in understanding chaotic behavior in psychological and social systems. Again there's a lot of symmetries, there's a lot of recognizable features. We don't have a verbal language for them; we are trying to develop a visual language for the recognizable features of these patterns, using currently the method of critical curves. That was invented by a mathematician in the south of France named Christian Ameira [sp?]. And we

have a few other ideas also, but basically here we are on the frontier of a new branch of mathematics, the study of which would probably be impossible without computer graphics. But here, unlike the massively parallel systems of the earlier videos, here we don't need a supercomputer, only an engineering workstation, such as—in fact our best one costs only $10,000. Okay, now we can turn up the lights. [audience applauds] Well, I think that's enough of me talking, and now I'd like to consider your questions, or answers or statements to each other. We could try to discuss whatever occurs to you. Yeah? [from audience -- *?]**
STARTING EDITING AGAIN.
I want to talk a little more about applications to social systems. First of all, there are small societies, like groups. There are particular models made of systems like this for interactive systems of a small number of elements. For example, there's a political model for a single nation. It's not necessarily a good one, but it's classical. To try to model the arms race, we combine two of them into a model for a system of two nations. This particular activity, arms-race modeling, is the first application of dynamical modeling to the social sciences that I know about.

It dates from World War I, when Louis Fry Richardson -- a Quaker and conscientious objector in World War I -- became an ambulance driver. He saw an awful lot of dead bodies, decided to dedicate his career to ending war and made a mathematical model, a really simple one, for the arms race. It was rejected for publication. He felt sure he could prevent World War II, but because the paper wasn't allowed to be published until 1968 we had to suffer World War II —- in his view, at any rate. Now although it was unpublished, a few people knew about it, and Gregory Bateson applied it to a model he called schismogenesis, i.e., to the development of schism in an individual mind or in a group. He wrote about it extensively. If you look up schismogenesis, the genesis of a schism, in the index of his first book *Steps Toward an Ecology of Mind,* **you'll see that he discussed it quite a bit in his different papers: applications to anthropological and psychiatric fields, models for alcoholism and so on.**

It seems clear that the qualitative behavior in one social arena is seen in a similar but slightly different form in another social sphere. We would expect that a decent mathematical or dynamical model for any social phenomenon could be applied over and over again in different spheres. We tried a model of three nations, and of course the three nations are in an arms race, and of course the behavior is chaotic. For two nations it tends to be periodic: as the arms in one nation escalate until they reach a certain limit, the other nation catches up while the first one goes down, and then you enter a disarmament cycle. As soon as there's a third player in the game, you get chaotic cycles with rapid drops, which are extremely expensive. For example, now they are dumping all the nuclear weapons in Europe, thank God, but maybe a few years later they'll start building them up again.

Basically, this is what happens: arms are dumped, then they are replaced. The utility of this cycle for the arms industry is quite obvious. If you have a very active market, you can keep manufacturing. If the arms industry could find a way to amplify these motions of a three-nation system, they'd be doing it subversively with the CIA or whatever, all over the globe. So the arms race was the first area where there was extensive development of a dynamical

model, and it immediately had a spinoff through Gregory Bateson to psychotherapy and social therapy.

If we could make decent dynamical models for social interactions and find a way to use them in therapy, we would eventually try to extend the model to more and more massive complex systems, and eventually we would attempt therapy on the planet at large. That's more or less what Jim Lovelock advocates in his latest book, *Healing Gaia.* **If a mathematical model to improve the relations between the genders could be devised, I think we'd be in good shape. And there is one. The book** *The Chalice and the Blade* **by Riane Eisler, published about five years ago, actually utilizes chaos theory in proposing an intentional intervention and producing a social transformation to restore us to the partnership society of the past.**

The question is, what is the relationship between the mathematical model and the therapy situation? In the mathematical models we are computing numbers. Let's say we had a catastrophic situation that looked like an alcoholic binge or something -– such a vague metaphor would not be very interesting. We would prefer to have something like sociometrics, psychometrics, some kind of more exact or qualitative relationship between the mathematical model and the observed psychological situation. It seems a little far-fetched, but this is actually a project I've been involved with for a few years, with a particular psychoanalytic group. Experimental sessions with consenting subjects were videotaped and the audio track was transcribed by a typist into very short lines. To score the phrases, ideas and words, we used a team of scorers who were psychiatrists trained in different disciplines who produced a gigantic spreadsheet of numbers which numerically tracked certain themes, like gender themes, displacement -– how far away in time or space is the thing being described from the immediate present in the therapy room –- things like that.

These huge spreadsheets had numbers, and we tried to subject them to the visualization technique to see if we can recognize shapes. So that's an ongoing project for which I don't have great hopes in the long run because it's so laborious to get the data. It's very expensive data. We've had about six sessions that have been scored many different ways and studied in every conceivable manner. Originally we imagined a kind of robot program based on speech recognition software where the scoring system would be an algorithm, a computer program that would respond to speech in more or less real time, so that the dynamical system could create a display on a computer screen, right on the video screen, as we speak.

Unless I tell you the details of the scoring system, this has nothing to do with psychotherapy. It's just what we might do in a conversation. We imagine, for example, Gorbachev and Reagan in a discussion over the future of the world. While they speak, there is a computer screen between them with two dots. We move chess pieces around, illustrating certain indicators extracted from their speech. On a telephone, we have an indicator where the red light goes on when the person is lying. And then we could see the two chess pieces moving around on the board. This fantasy is called "communicative chess". Now suppose that not only we could see it, but they could see it too. Then they'd have a chaoscope, mathematically-illuminated bio-feedback, socio-psycho-feedback, as it were, in their communication process and they could see that somebody was trying to trap them in a corner and then, according to some end game invented by Tricky Dick Nixon, they'd know how to get out.

Well, this is just an idea. I don't know if it will succeed in the long run, but keep in mind that the computer technology to do this is available. It's still expensive now, but soon it will be cheap, and then somebody is going to do this, and maybe it had better be us. To come back to the arms race model, Louis Fry Richardson thought up an imaginary model that was vaguely realistic. Today we wouldn't think it was very good. He supposed that one of the main variables would be the total amount of armament. Another important parameter would be some kind of psychological state of paranoia, a populace more or less fearful of an invasion by the neighbors or somebody. These were his main variables. When he submitted this for publication and it was rejected, he revised it and submitted it again. The model didn't change very much, so he tried to defend it by comparing it with data. In order to do this, he invented a new field, now called political metrics. He studied historical records about the armaments of different nations engaged in war starting around the year 1700. He estimated the number of deaths in the battlefield and compared it to the population, trying to get a number which he called the "cost" of a war. Then he tried to numerically parameterize the paranoia of a society, according to their perception of the cost of a war if it were to happen. But it didn't happen yet, and so he had a model which could be compared with data that didn't really exist but could be extracted from the historical record.

This activity is still going on in political science. For example, at the University of Michigan there's a project called the Correlates of War. They have 100 parameters to measure from the historical records now, and the models for war are getting better and better. We would like to have models for peace. There is a well-established, superb information theory in the field of chaos. In fact, there's an excellent pedagogic book, a textbook called *The Dripping Faucet* **as a Model of Chaotic System by Robert Shaw. According to his view, chaotic systems create information, and the more chaotic they are the faster information is being created. The rate of creation of information is a measure of how chaotic or complex the system is. I think however that this particular aspect of chaos theory will not be such an important one in social science applications. The intuitive feeling we have that time is accelerating —- information, chaos, complexity is accelerating -— probably has to do with the disintegration of society but needn't necessarily be visible in a mathematical model.**

Is there a possibility of modeling presidential races? This is actually a very exciting field, but I think it's not quite ready for application. There is in the social sciences a field called Social Choice Theory. It emerged first in mathematical economics when they tried to estimate the market for a certain product. "How many people would choose this, and for what reason, and why do people choose to invest in this stock and not that one?" and so on. Out of that came a whole mathematical theory of voting, which is, as a matter of fact, the current frontier to predict three-way races. One thing that's not in the model, yet might be the most important thing in this case, is the variable of franchise: how many people will vote? The choice to vote or not to vote might be crucial in this particular case, and the modeling didn't develop to that level yet. Maybe by the next race. It's a very active field in which really brilliant and well-prepared people are working.