Statistical Mechanics of Networks

TROISIEME CYCLE DE PHYSIQUE EN LA SUISSE ROMANDE

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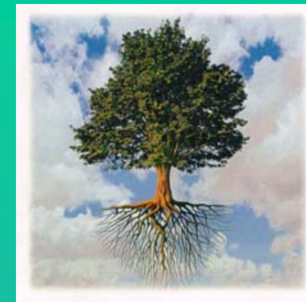
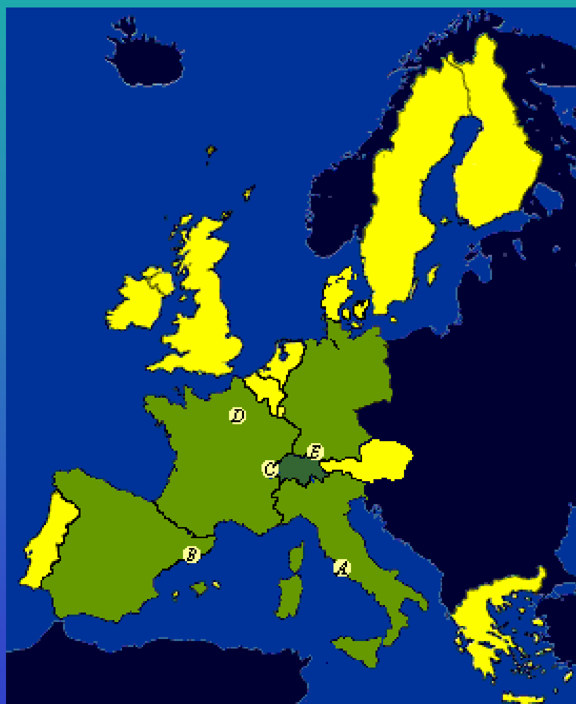
•Contents

COSIN
COevolution and Self-organisation In dynamical Networks



FET Open scheme RTD Shared Cost Contract IST-2001-33555

http://www.cosin.org



- **Nodes** 6 in 5 countries
- **Period of Activity**: April 2002-April 2005
- **Budget:** 1.256 M€
- **Persons financed**: 8-10 researchers
- **Human resources**: 371.5 Persons/months

EU countries

Non EU countries

EU COSIN participant

Non EU COSIN participant

•2 Boring stuff (1/3)

- The graph **size** is the number of its vertices.
- The graph **measure** is the number of its edges.
- The **degree** of a vertex in a graph is the number of edges that connects it to other vertices.
- In the case of an oriented graph the degree can be distinguished in **in-degree** and **out-degree**.
- Whenever all the vertices share the same degree the graph is called **regular**.
- A series of consecutive edges forms a **path**.
  - The number of edges in a path is called the length of the path.
  - A Hamiltonian path is a path that passes once through all the vertices (not necessarily through all the edges) in the graph.
  - A Hamiltonian cycle is a Hamiltonian path which begins and ends in the same vertex.
  - An Eulerian path is a path that passes once through all the edges (not necessarily once through all the vertices) in the graph.
  - An Eulerian cycle is an Eulerian path which begins and ends in the same edge.

•2 Boring stuff (2/3)

- Whenever all the vertices share the same degree the graph is called **regular**.
- A series of consecutive edges forms a **path**.
  - oThe number of edges in a path is called the length of the path.
  - oA Hamiltonian path is a path that passes once through all the vertices (not necessarily through all the edges) in the graph.
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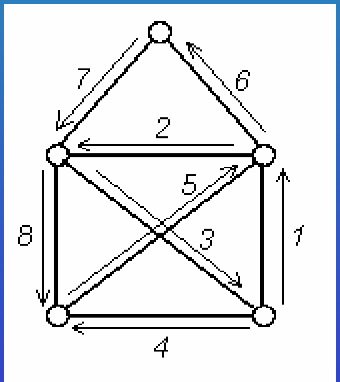
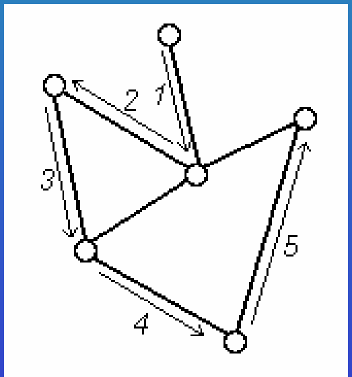
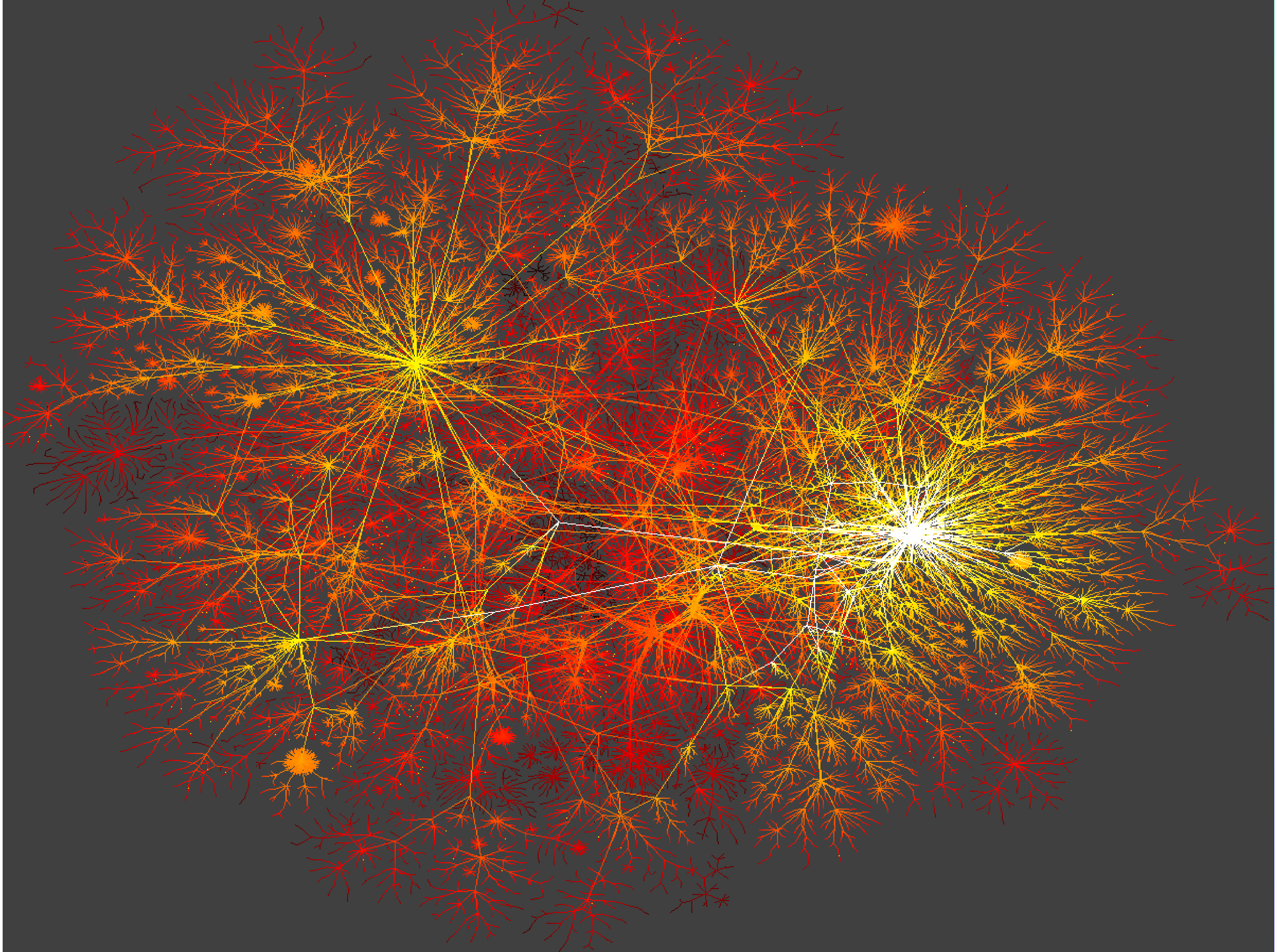


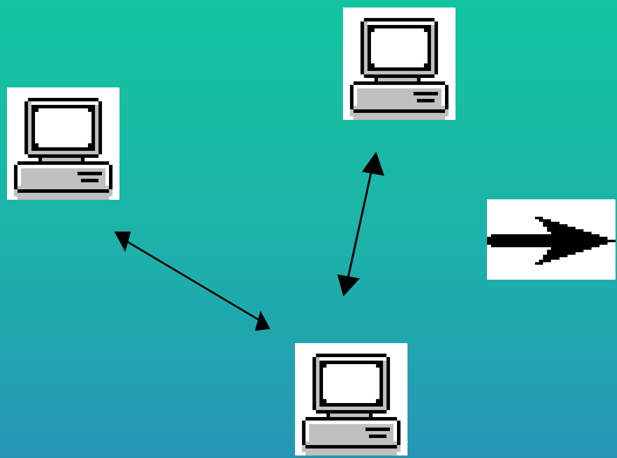


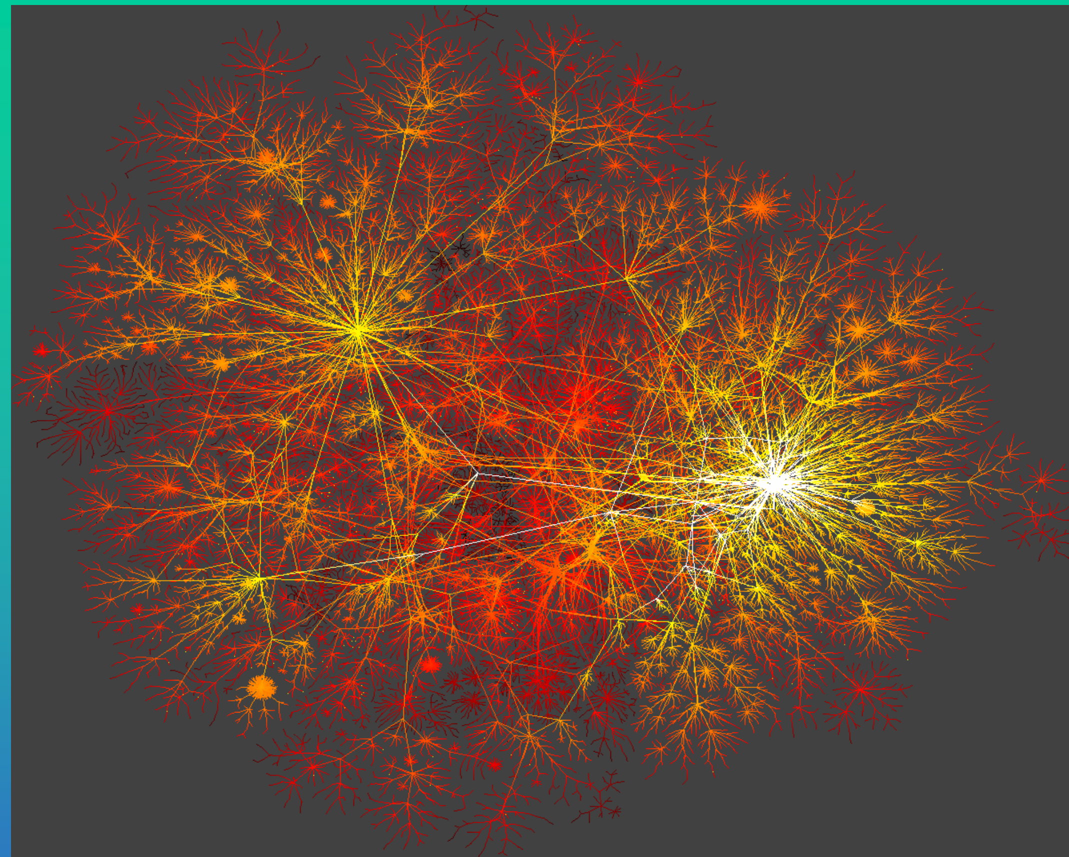
•2 Boring stuff (3/3)

- A graph is **connected** if a path exists for any couple of vertices in the graph.
- A graph with no cycles is a **forest**. A **tree** is a connected forest.
- The **distance** between two vertices is the shortest number of edges one needs to travel to get from one vertex to the other.
- Therefore the **neighbours** of a vertex are all the vertices which are connected to that vertex by a single edge.
- A **dominating set** for a graph is a set of vertices whose neighbours, along with themselves, constitute all the vertices in the graph.
- A graph with size n cannot have a measure larger than $mmax = n(n-1)/2$. When all these possible edges are present the graph is **complete** and it is indicated with the symbol Kn.
- The opposite case happens when there are no edges at all. The measure is 0 and the graph is then **empty** and it is indicated by the symbol En.
- The **diameter** D of a graph is the longest distance you can find between two vertices in the graph.
- A complete **bipartite clique** Ki,j is a graph where every one of i nodes has an edge directed to each of the j nodes.
- The **clustering coefficient** C is a rougher characterization of clustering with respect to the clique distribution. C is given by the average fraction of pair of neighbours of a node that are also neighbours each other. For an empty graph En $C=0$ everywhere. For a complete graph Kn, $C=1$ everywhere.
- A **bipartite core** Ci,j is a graph on $i+j$ nodes that contains at least one Ki,j as a subgraph.



•2A Internet





Router connections at small level produce a **complex** Internet structure.

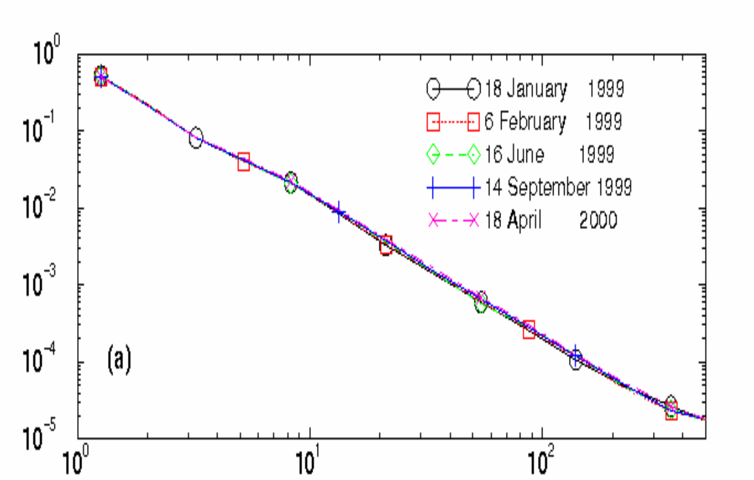
•2A Internet

Previous maps have been computed through extensive collection of traceroutes

gcalda@pil.phys.uniroma1.it> traceroute www.louvre.fr

1	141.108.1.115	Rome	pcpil
2	141.108.5.4	Unknown	
3	193.206.131.13	Unknown	rc-infnrmi.rm.garr.net
4	193.206.134.161	Unknown	rt-rc-1.rm.garr.net
5	193.206.134.17	Unknown	mi-rm-1.garr.net
6	212.1.196.25	South Cambridgesh	garr.it.ten-155.net
7	212.1.192.37	South Cambridgesh	ch-it.ch.ten-155.net
8	212.1.194.14	Genève	geneva5.ch.eqip.net
9	195.206.65.105	Genève	geneva1.ch.eqip.net
10	0.0.0.0	Unknown	No Response
11	193.251.150.30	Unknown	p6.genar2.geneva.opentransit.net
12	193.251.154.97	PARIS, FR	p43.bagbb1.paris.opentransit.net

•2A Internet

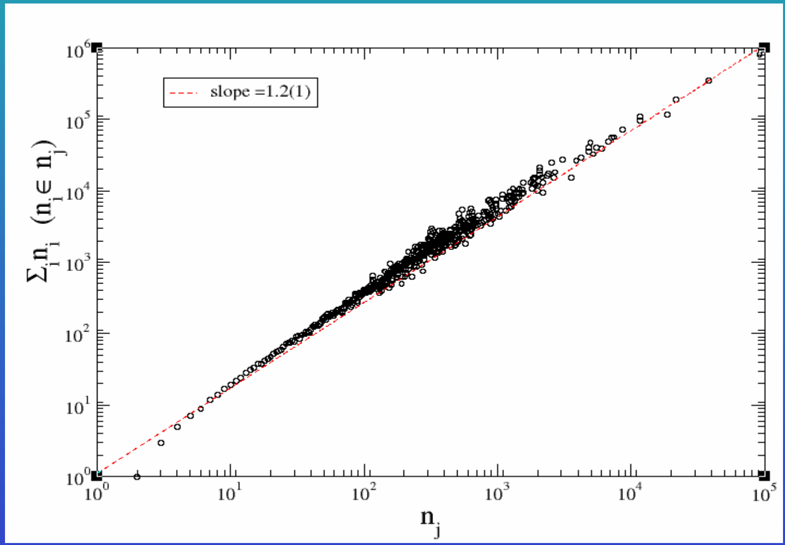


Results are that we can quantify the hierarchical nature of the AS connections

$$P(A) \propto A^{-2}$$

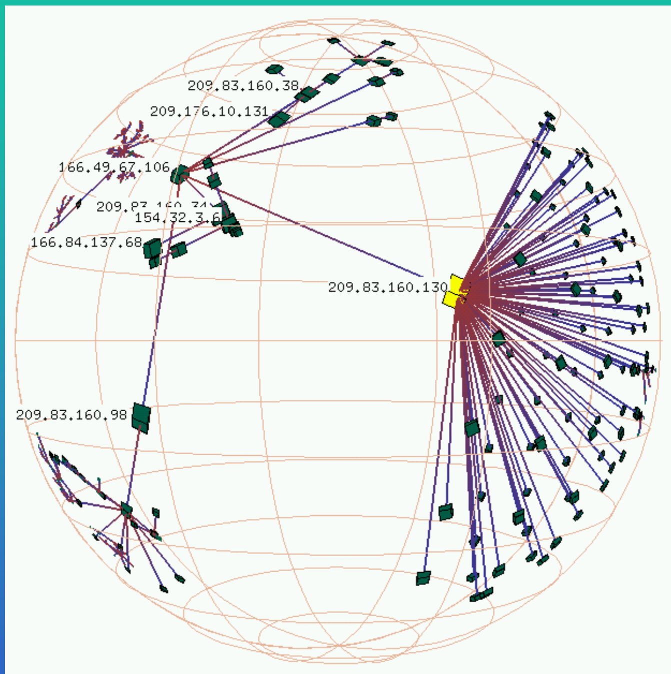
Plot of the C(A) show the same optimisation of the Food webs

$$C(A) \propto A$$



•2A Internet

skitter is a tool for actively probing the Internet in order to analyze topology and performance.



•Measure Forward IP Paths
skitter records each hop from a source to many destinations. by incrementing the "time to live" (TTL) of each IP packet header and recording replies from each router (or hop) leading to the destination host.

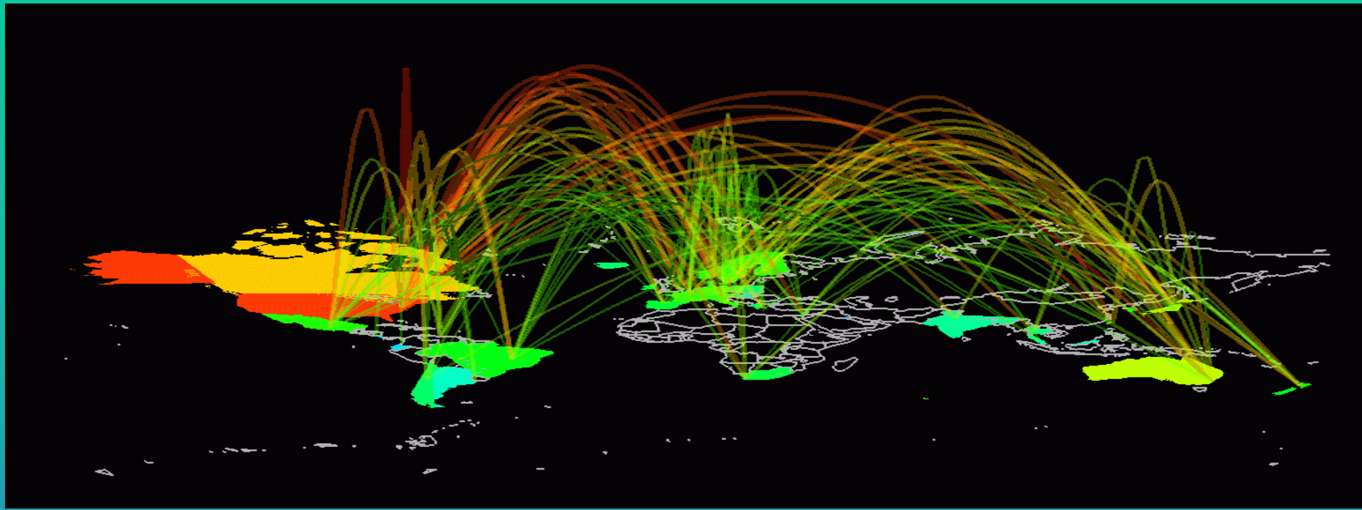
•Measure Round Trip Time
skitter collects round trip time (RTT) along with path (hop) data. skitter uses ICMP echo requests as probes to a list of IP destinations.

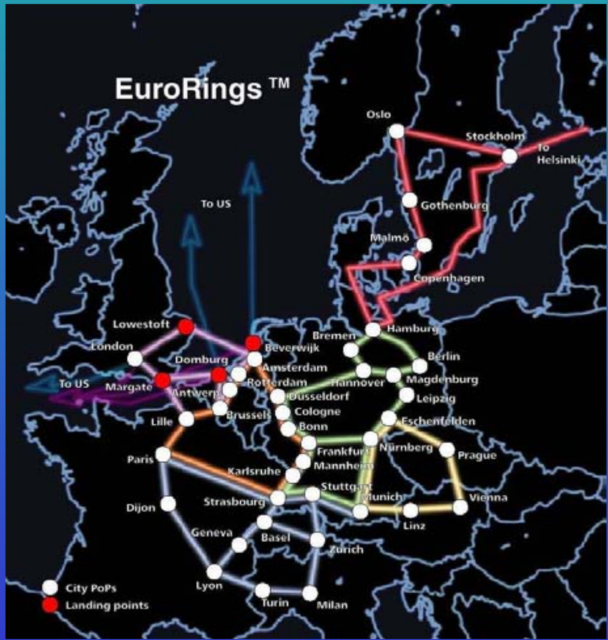
•Track Persistent Routing Changes
skitter data can provide indications of low-frequency persistent routing changes. Correlations between RTT and time of day may reveal a change in either forward or reverse path routing.

•Visualize Network Connectivity
By probing the paths to many destinations IP addresses spread throughout the IPv4 address space, skitter data can be used to visualize the directed graph from a source to much of the Internet.

http://www.caida.org/tools/measurements/skitter

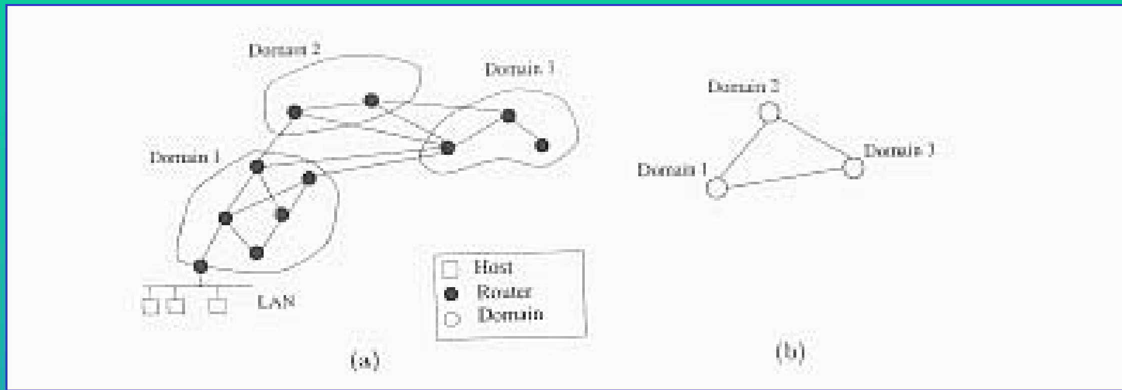
•2A Internet




EuroRings ™
Oslo
Stockholm
To Helsinki
To US
Gothenburg
Malmö
Copenhagen
Lowestoft
London
Beverwijk
Bremen
Hamburg
Domburg
Amsterdam
Berlin
To US
Margate
Antwerp
Rotterdam
Hannover
Magdeburg
Dusseldorf
Leipzig
Lille
Brussels
Cologne
Bonn
Eschenfelden
Paris
Frankfurt
Nürnberg
Prague
Mannheim
Karlsruhe
Stuttgart
Strasbourg
Munich
Vienna
Dijon
Linz
Geneva
Basel
Zurich
Lyon
Turin
Milan
City PoPs
Landing points


OSLO
STOCKHOLM
HELSINKI
NORTH SEA
RIGA
MOSCOW
COPENHAGEN
DUBLIN
LONDON
AMSTERDAM
WARSAW
EUROPE
BRUSSELS
BONN
KIEV
PARIS
ZAGREB
BUCHAREST
BLACK SEA
SOFIA
TIRANA
MADRID
ANKARA
LISBON
ALGIERS
TUNIS
ATHENS
NICOSIA
BEIRUT
DAMASCUS
RABAT
TRIPOLI
TEL AVIV
CAIRO

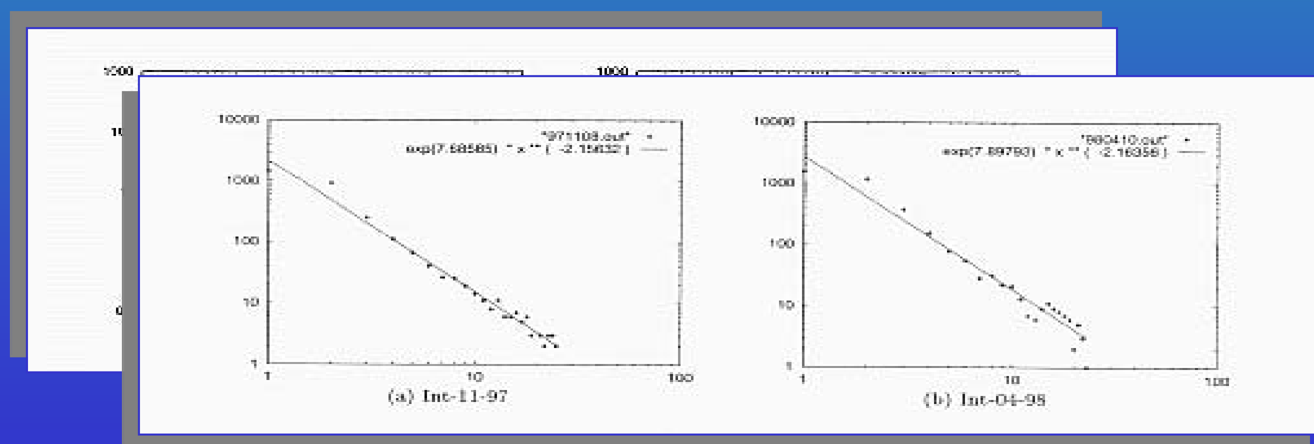
•2A Internet

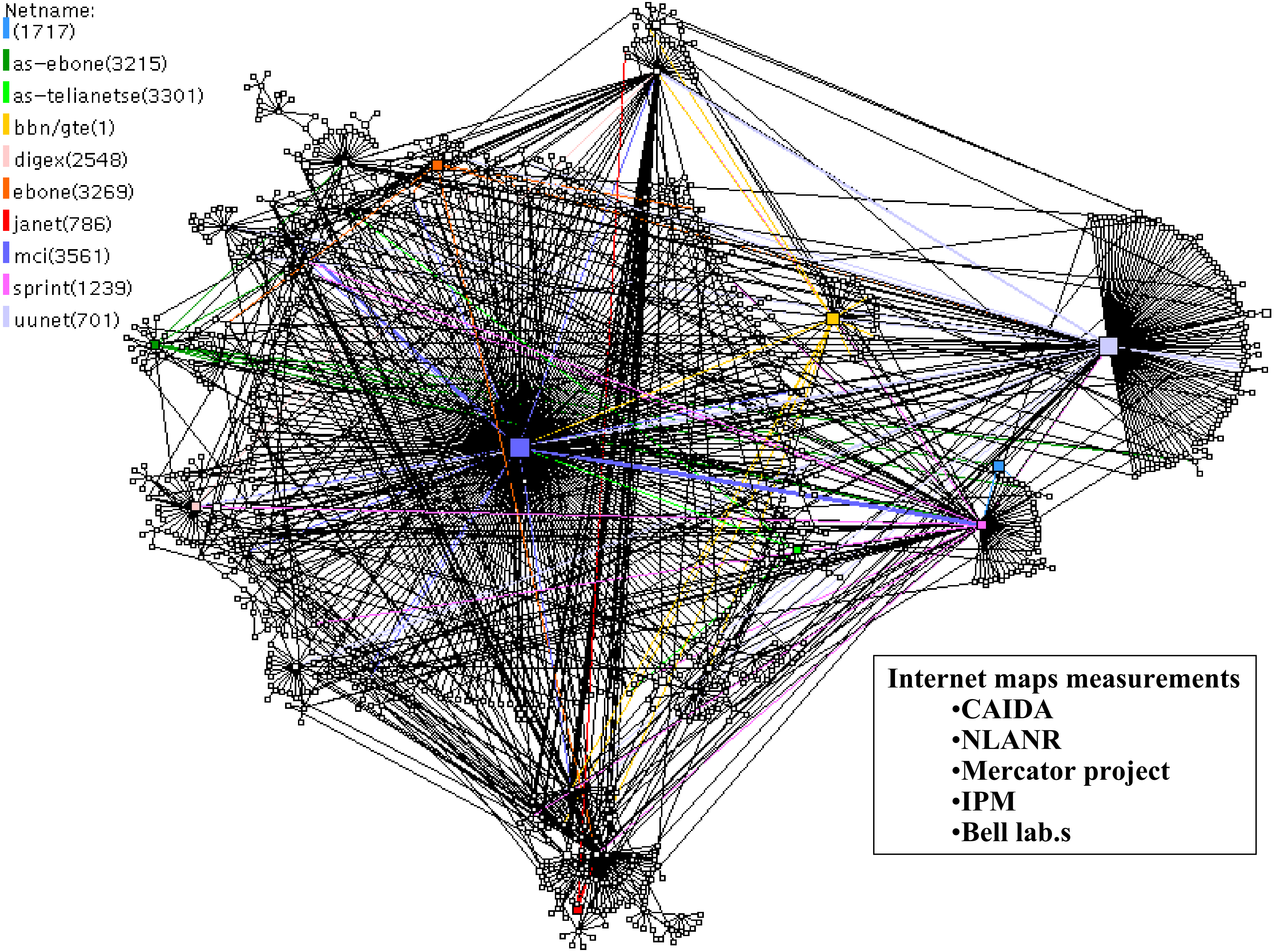


This happens at both domain and router server

•*P(k)* = probability that a node has *k* links

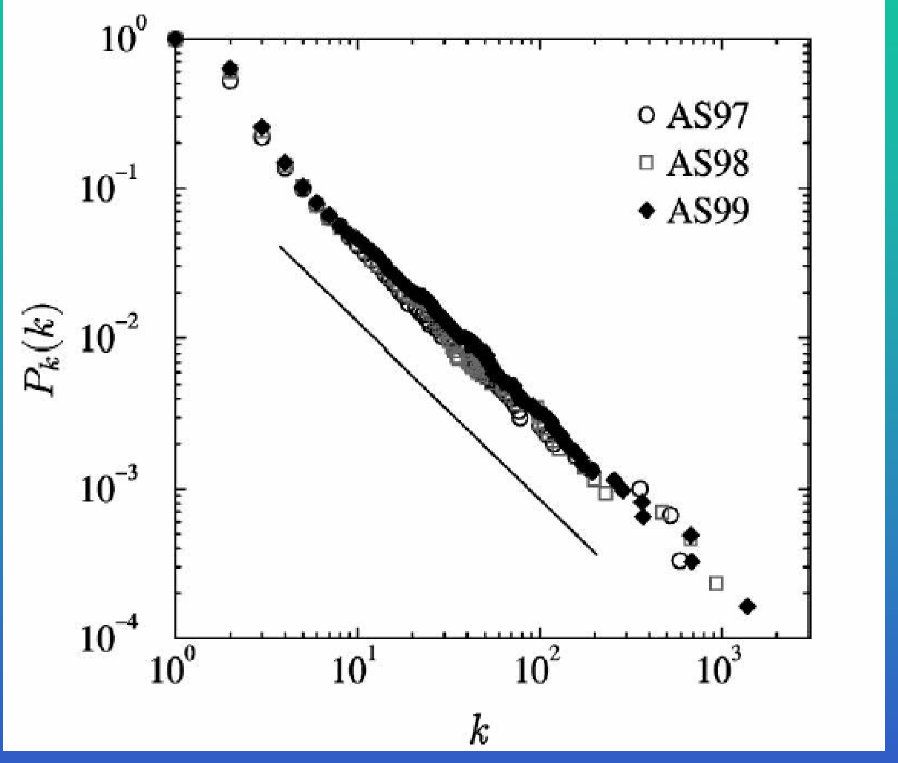
Faloutsos et al. (1999)





Internet maps measurements
- **CAIDA**
- **NLANR**
- **Mercator project**
- **IPM**
- **Bell lab.s**

•2A Internet



Vazquez Pastor-Satorras and Vespignani
PRE 65 066130 (2002)

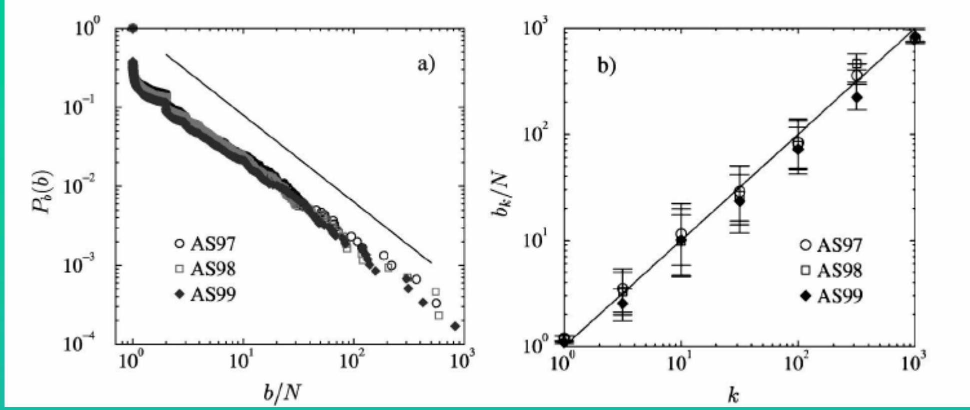
TABLE I. Total number of new (N_{new}) and deleted (N_{del}) nodes in the years 1997, 1998, and 1999. We also report the number of deleted nodes with connectivity $k > 10$.

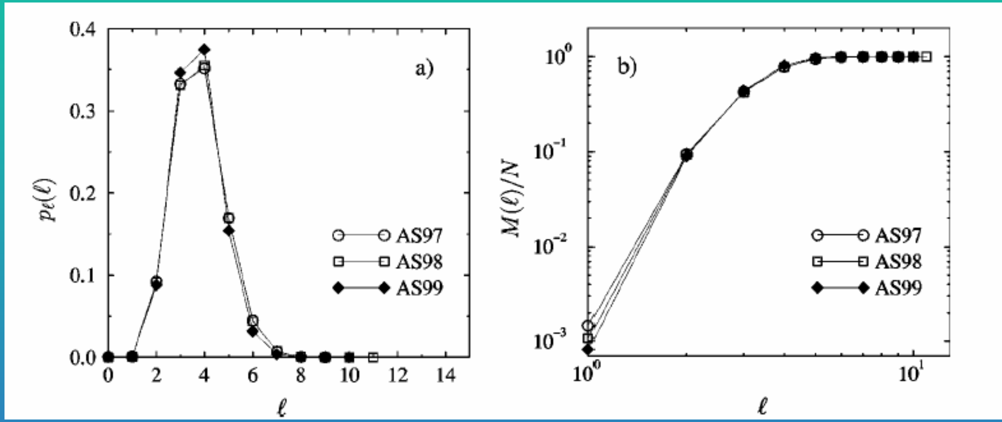
Year	1997	1998	1999
N_{new}	309	1990	3410
N_{del}	129	887	1713
$N_{del}(k>10)$	0	14	68

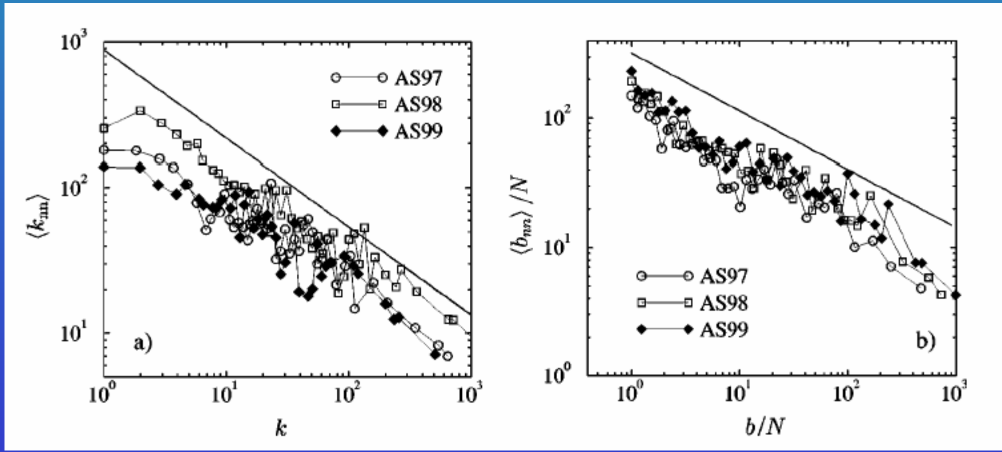
TABLE II. Average properties of the Internet for three different years. N, number of nodes; E, number of connections; $\langle k \rangle$, average connectivity; $\langle c \rangle$, average clustering coefficient; $\langle \ell \rangle$, average path length; $\langle b \rangle$, average betweenness. Figures in parentheses indicate the statistical uncertainty from averaging the values of the corresponding months in each year.

Year	1997	1998	1999
N	3112	3834	5287
E	5450	6990	10100
$\langle k \rangle$	3.5(1)	3.6(1)	3.8(1)
$\langle c \rangle$	0.18(3)	0.21(3)	0.24(3)
$\langle \ell \rangle$	3.8(1)	3.8(1)	3.7(1)
$\langle b \rangle / N$	2.4(1)	2.3(1)	2.2(1)

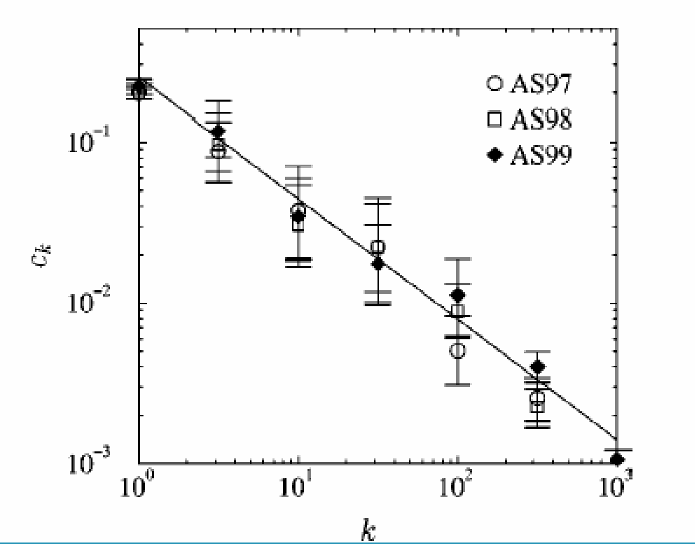
•2A Internet


a)
b)
$P_b(b)$
b/N
AS97
AS98
AS99
b_k/N
k

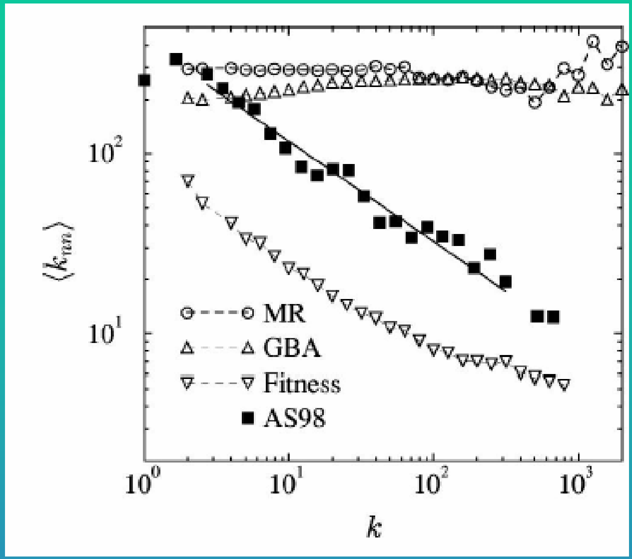

a)
b)
$p_\ell(\ell)$
$M(\ell)/N$
ℓ
AS97
AS98
AS99


a)
b)
$\langle k_{nn} \rangle$
$\langle b_{nn} \rangle /N$
k
b/N
AS97
AS98
AS99

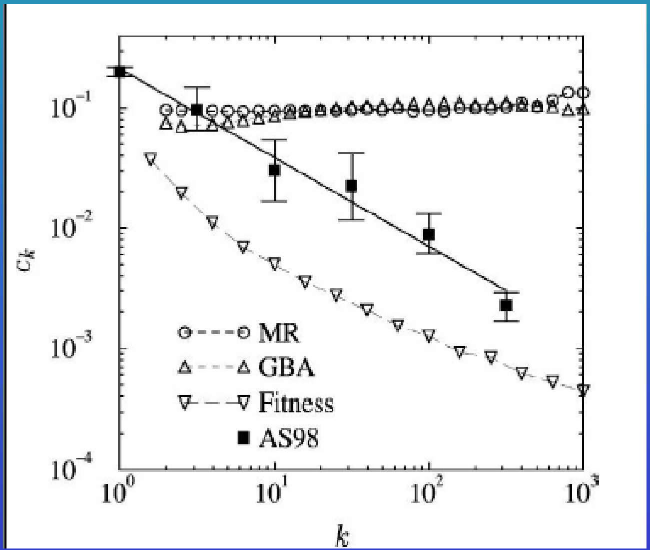
•2A Internet



AS97
AS98
AS99
c_k
10^{-1}
10^{-2}
10^{-3}
10^0
10^1
10^2
10^3
k

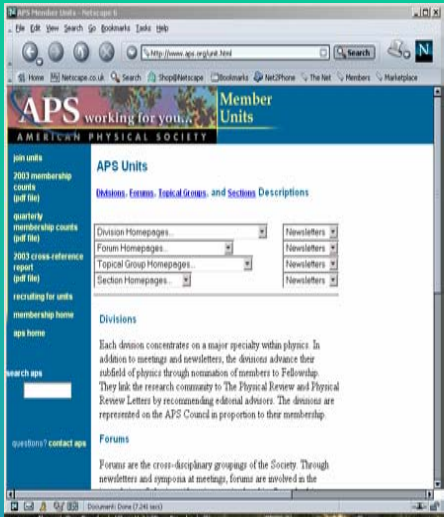


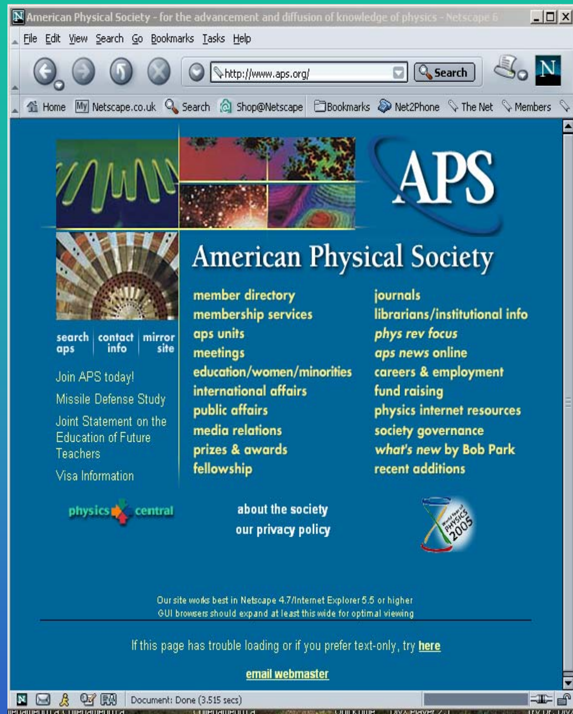
$\langle k_{nn} \rangle$
10^2
10^1
MR
GBA
Fitness
AS98
10^0
10^1
10^2
10^3
k

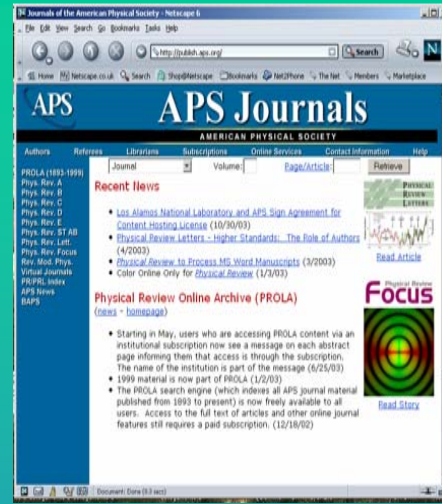


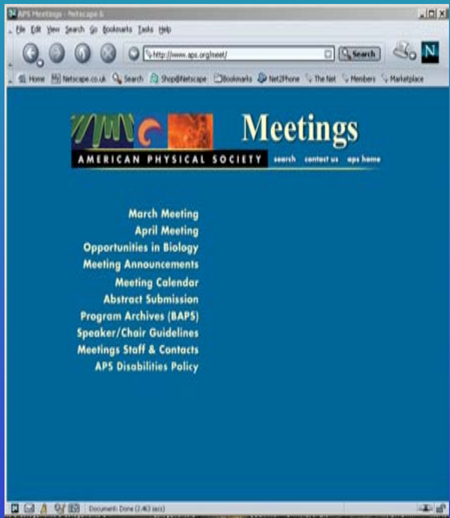
c_k
10^{-1}
10^{-2}
10^{-3}
10^{-4}
MR
GBA
Fitness
AS98
10^0
10^1
10^2
10^3
k

•2B World Wide Web









Nodes: (static) HTML pages

Edges (directed): hyperlinks beetween pages

Why are we interested in the WebGraph?

From link analysis:

- Data mining (ex: PageRank)
- Sociology of content creation
- Detection of communities

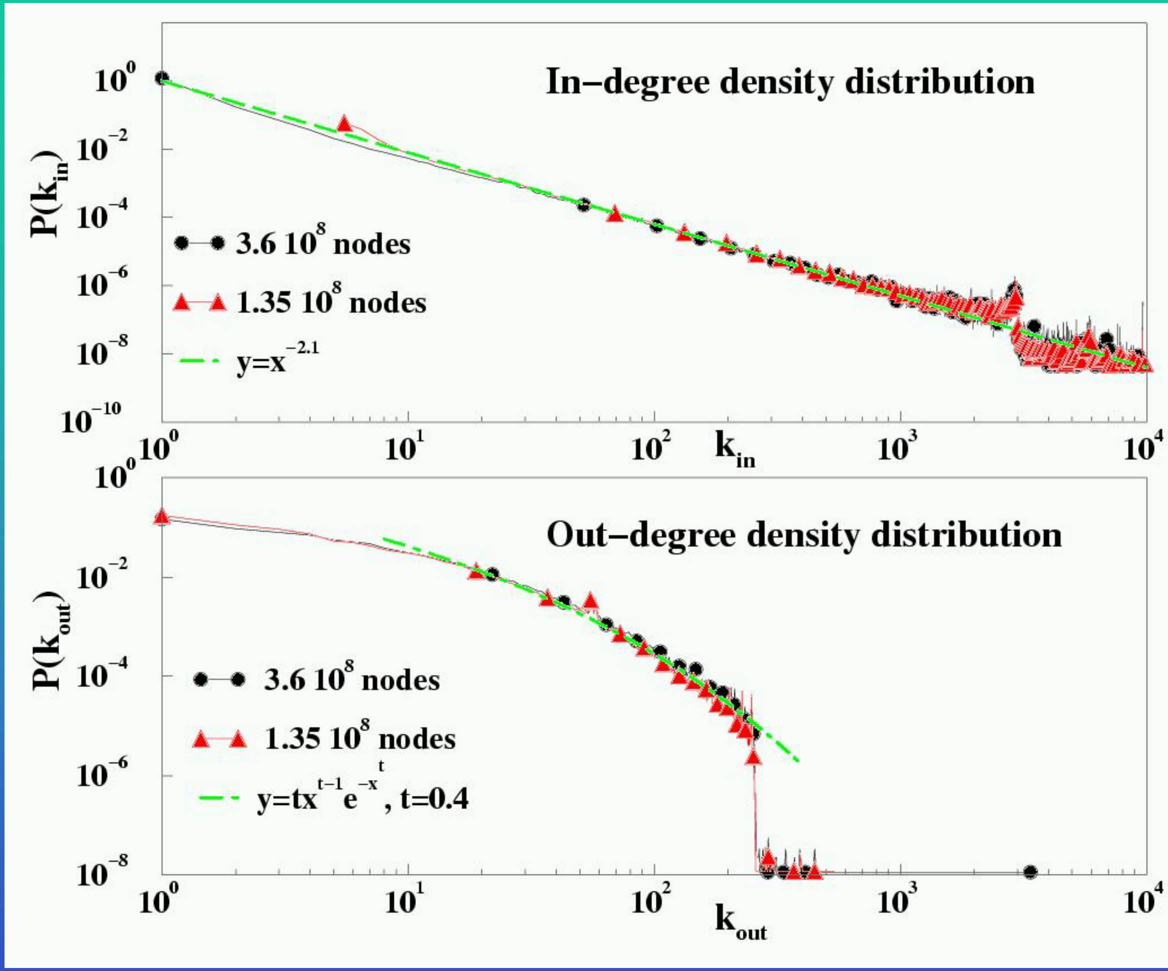
With a “good” WebGraph model:

- Prove formal properties of algorithms
- Detect peculiar region of the WebGraph
- Predict evolution of new phenomena

Models for the WebGraph:

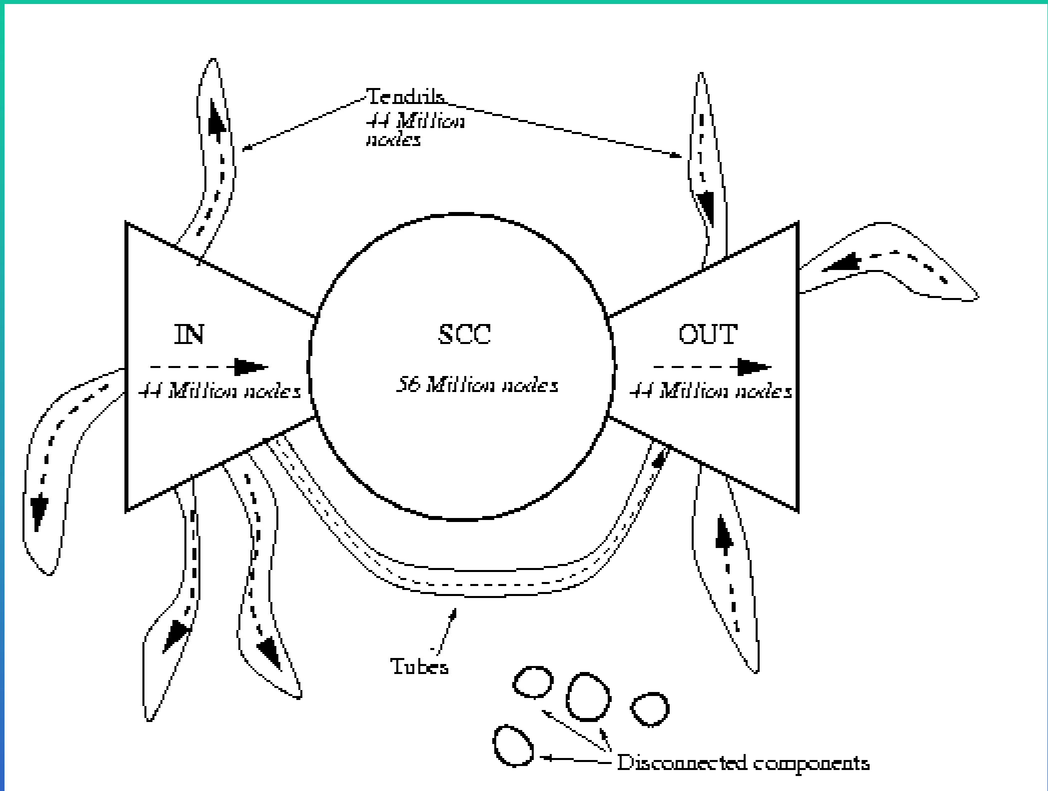
- Random Graph (Erdös, Renyi)
- Evolving networks (Albert, Barabasi, Jeong)
- “Copying” models (Kumar, Raghavan,…)
- ACL for massive graph (Aiello, Chung, Lu)
- Small World (Watts, Strogats)
- Fitness (Caldarelli, Capocci, De Los Rios, Munoz)
- Multi-Layer (Caldarelli, De Los Rios, Laura, Leonardi)

•2B World Wide Web



Albert Barabasi ***Emergence of scaling in random networks***
Kumar et al. , ***Stochastic models for the WebGraph***
Broder et al. , ***Graph structure in the web***

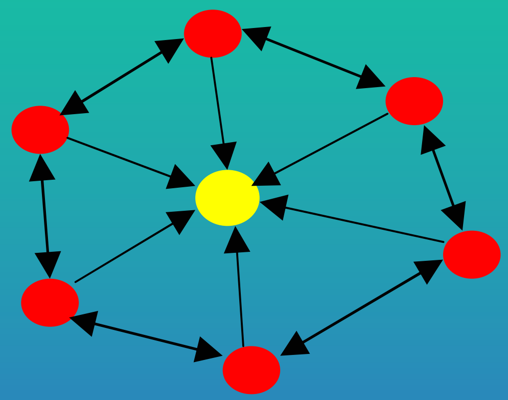
•2B World Wide Web



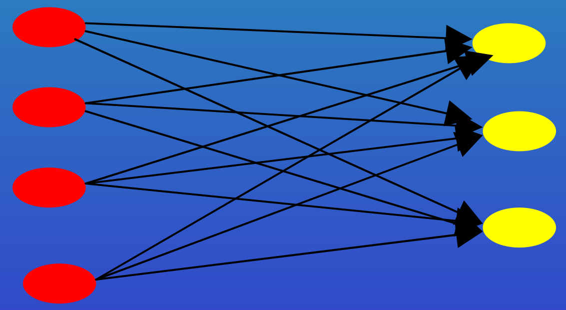
- Bow-tie structure
- Small World for the SCC and the weakly connected components

Broder et al. , ***Graph structure in the web***

Cyber Communities



- **Explicit** (or “self-aware”) communities:
  1. Webrings
  2. Newsgroup users
  3. Gnutella, Morpheus, etc.. users
- **Implicit** communities:
  1. Fan-Center Bipartite Cores



Kumar et al. , ***Crawling the Web for Emerging Cyber Communities***

Fractal properties

- **TUC** - Thematically Unified Cluster, for example:

1. By content
2. By location
3. By geographical location

…and…

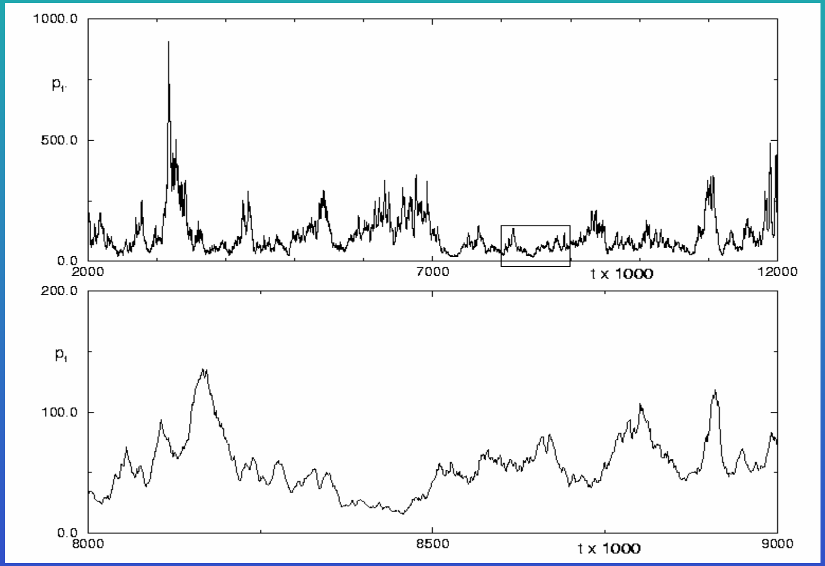
4. Random collection of websites
5. Hostgraph

Dill et al. , *Self-similarity in the web*

•2C Economics and Finance

Probably the most complex system is human behaviour!

Even by considering only the trading between individuals, situation seem to be incredibly complicated.



Econophysics tries to understand the basic “active ingredients” at the basis of some peculiar behaviours.

For example price statistical properties can be described through a simple model of agents trading the same stock.

“A Prototype Model of Stock Exchange”

Europhysics Letters, **40** 479 (1997), G. C., M. Marsili, Y.-C. Zhang.

•2C Economics and Finance

Some of the phenomena in finance can be described by means of graphs

• Stock price correlations

- J.-P. Onnela, A. Chackraborti, K. Kaski, J. Kertész, A. Kanto
  http://xxx.lanl.gov/abs/cond-mat/0303579 and *http://xxx.lanl.gov/abs/cond-mat/0302546*
- G. Bonanno, G. Caldarelli, F. Lillo and R. N. Mantegna
  http://xxx.lanl.gov/abs/cond-mat/0211546

•Portfolio composition

- D. Garlaschelli, S. Battiston, M. Castri, V. D. P. Servedio, G. Caldarelli
  http://xxx.lanl.gov/abs/cond-mat/0310503

•Board of Directors

- M. E. J. Newman, S. H. Strogatz and D. J. Watts,
  Phys. Rev. E **64**, 026118 (2001).
- S. Battiston, E. Bonabeau and G. Weisbuch
  http://xxx.lanl.gov/abs/cond-mat/0209590 (2002).

Through this new description we can

•Discover new features
•Validate Models

•2C Stock Correlations

$$r_i(\tau) = \ln P_i(\tau) - \ln P_i(\tau - 1)$$

Logarithmic return of stock *i*

$$\rho_{i,j} = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\left(\langle r_j^2 \rangle - \langle r_j \rangle^2\right)\left(\langle r_i^2 \rangle - \langle r_i \rangle^2\right)}}$$

Correlation between returns (averaged on trading days)

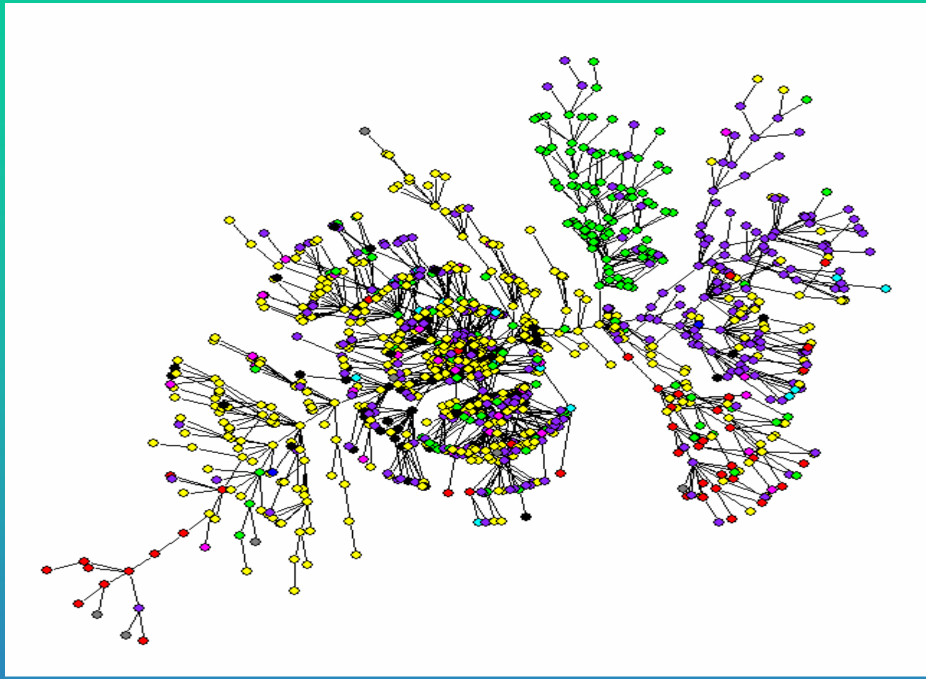
$$d_{i,j} = \sqrt{2(1 - \rho_{i,j})}$$

Distance between stocks *i, j*

A tree (a graph with no cycle) can be constructed by imposing that the sum of the (N-1) distances is the minimum one.

•2C Stock correlation

Real Data from NYSE



Correlation based minimal spanning trees of real data from daily stock returns of 1071 stocks for the 12-year period 1987-1998 (3030 trading days). The node colour is based on Standard Industrial Classification system.
The correspondence is:

red for mining
cyan for construction
yellow for manufacturing
green for transportation, communications, electric,gas and sanitary services
light blue for public administration
magenta for wholesale trade
black for retail trade
purple for finance and insurance
orange for service industries

"**Topology of correlation based..**" http://xxx.lanl.gov/abs/cond-mat/0211546

G. Bonanno, G. C. , F. Lillo, R. Mantegna.

•2C Stock correlation

Data from Capital Asset Pricing Model

In the model it is supposed that returns follow

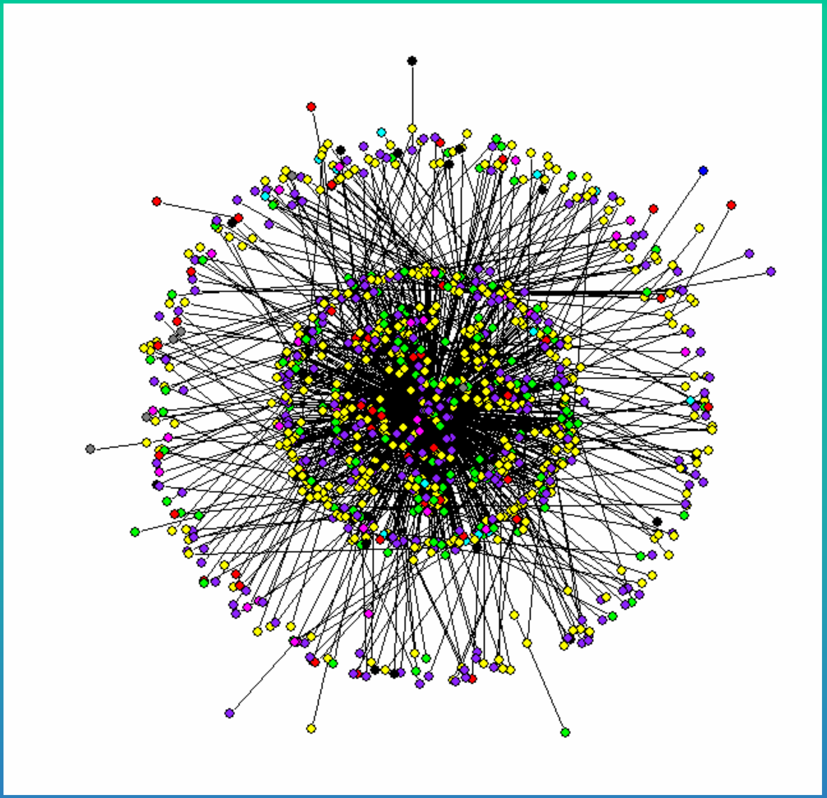
$$r_i(t) = \alpha_i + \beta_i r_M(t) + \varepsilon_i(t)$$

$r_i(t)$ = return of stock i

$r_M(t)$ = return of market (Standard & Poor's)

α_i, β_i = real parameters

ε_i, = noise term with 0 mean

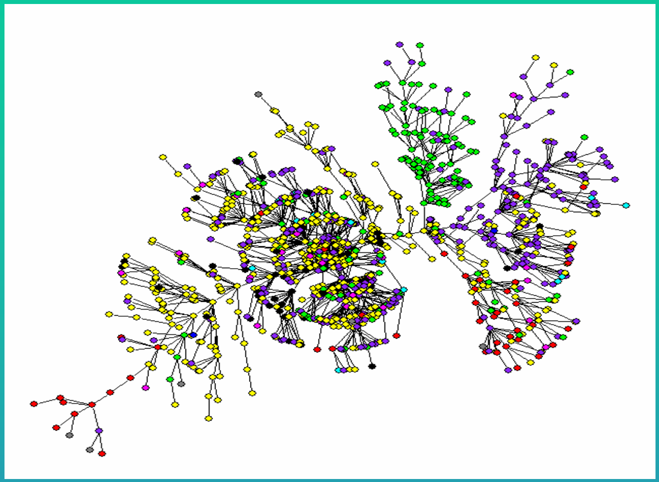


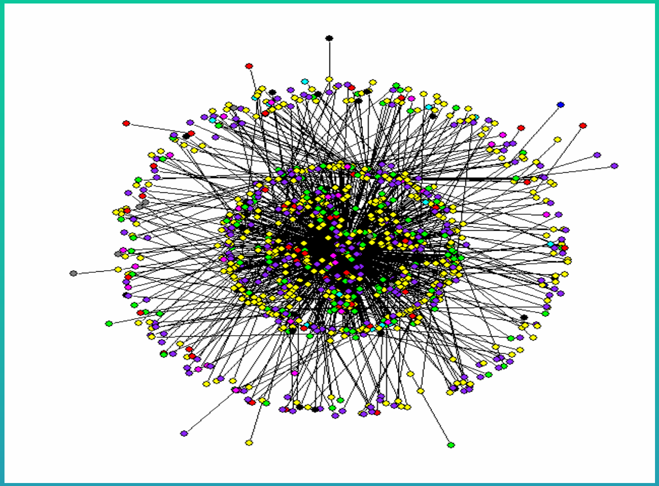
Correlation based minimal spanning trees of of an artificial market composed by of 1071 stocks according to *the one factor model*.

The node colour is based on Standard Industrial Classification system. The correspondence is:

red for mining
cyan for construction
yellow for manufacturing
green for transportation, communications, electric,gas and sanitary services
light blue for public administration
magenta for wholesale trade
black for retail trade
purple for finance and insurance
orange for service industries

•2C Stock correlation





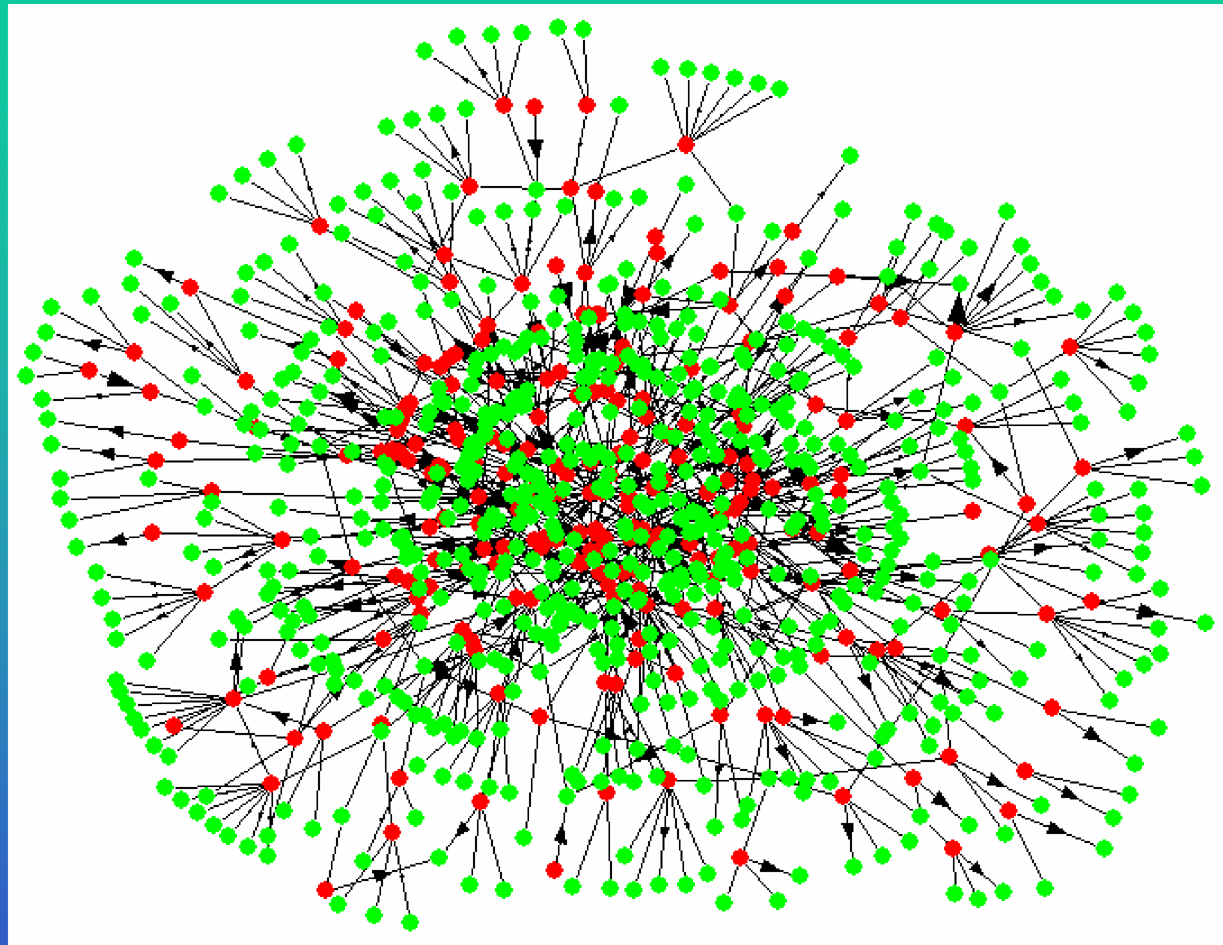
Without going in much detail about degree distribution or clustering of the two graphs

We can conclude that:

the topology of MST for the real and an artificial market are greatly different.

Real market properties are not reproduced by simple random models

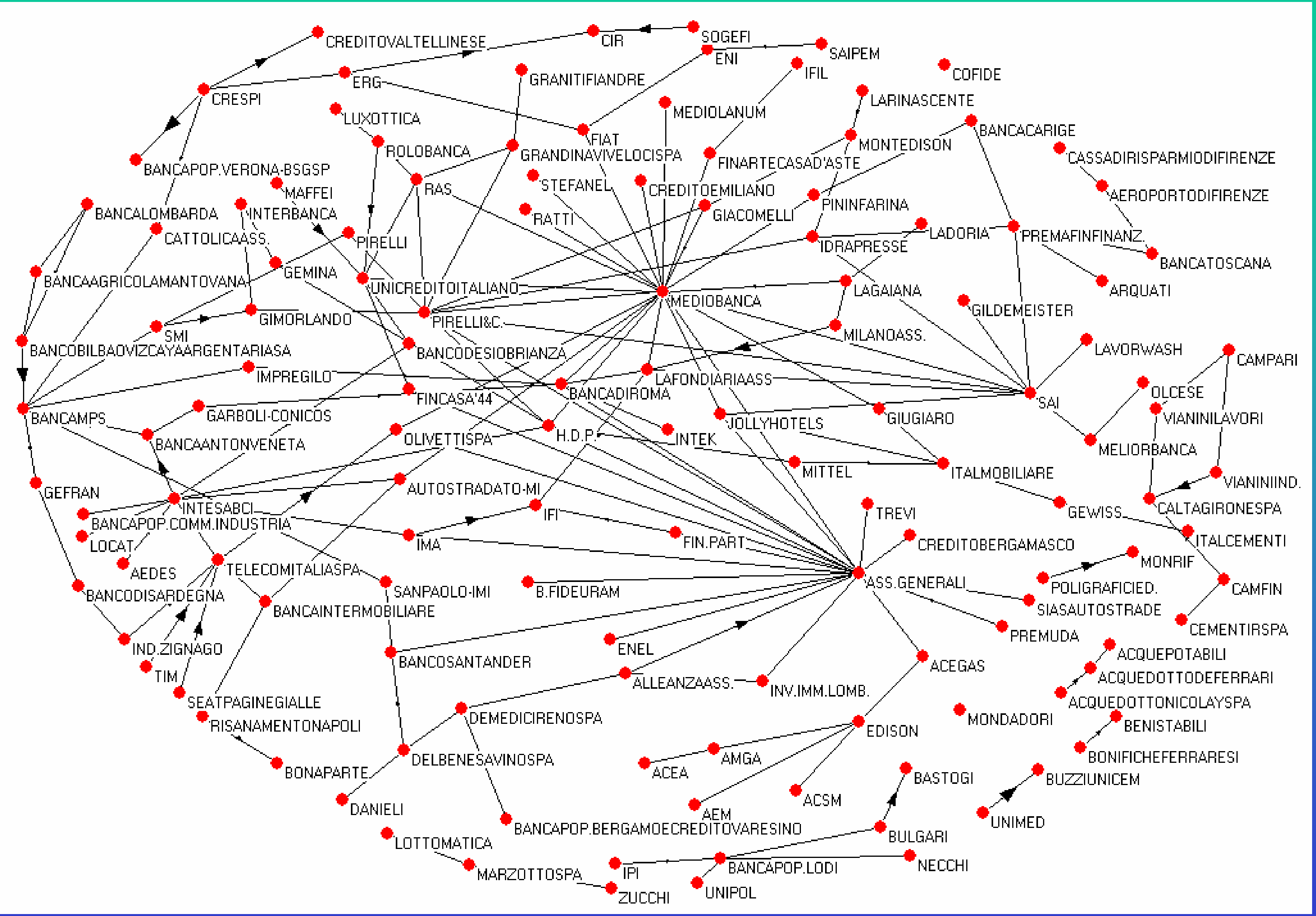
•2C Portfolio Composition



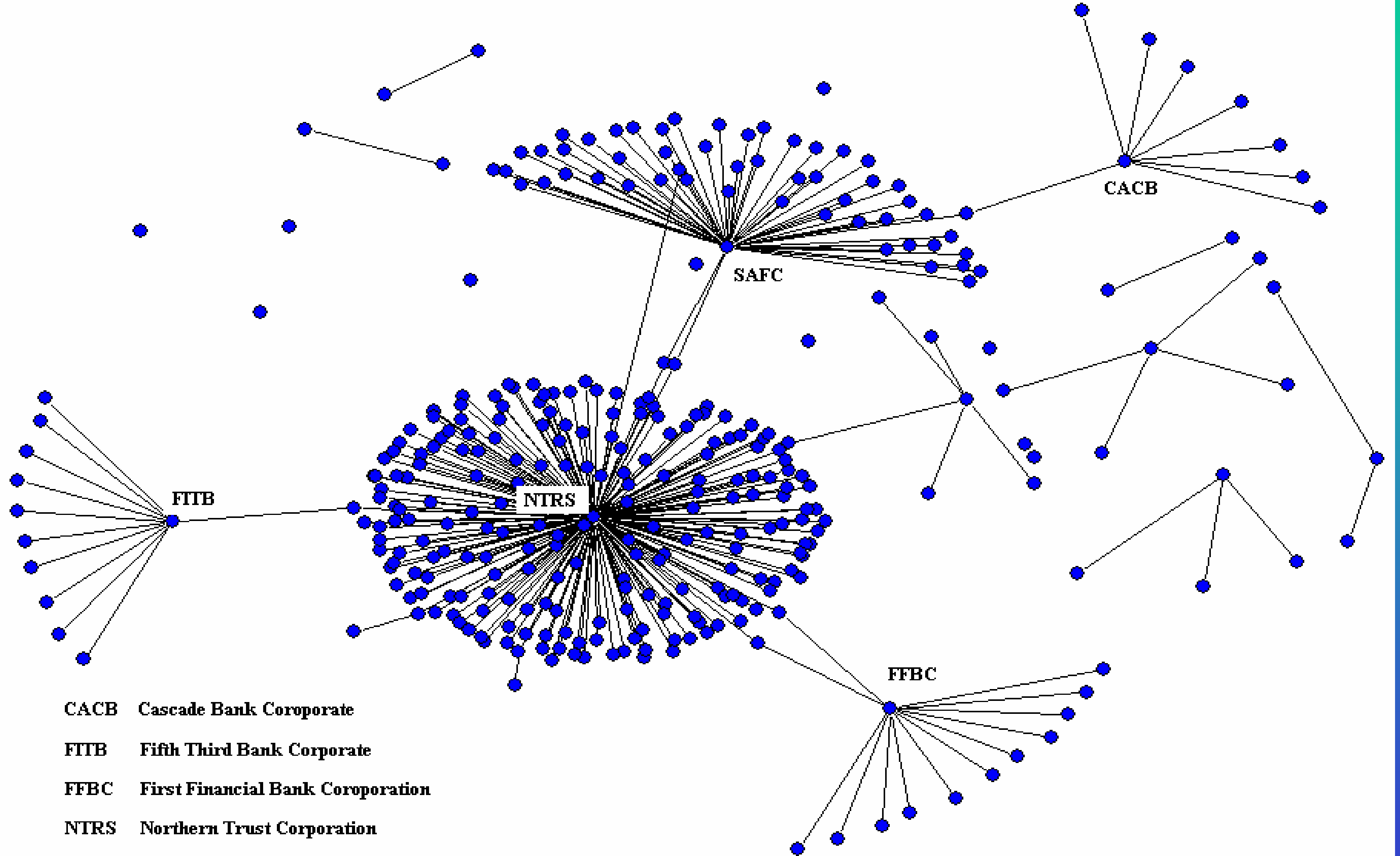
Investors or Companies not traded at Borsa di Milano (Italy)

Companies traded at Borsa di Milano (Italy)

•2C Portfolio Composition



•2C Portfolio Composition



CACB Cascade Bank Coroporate

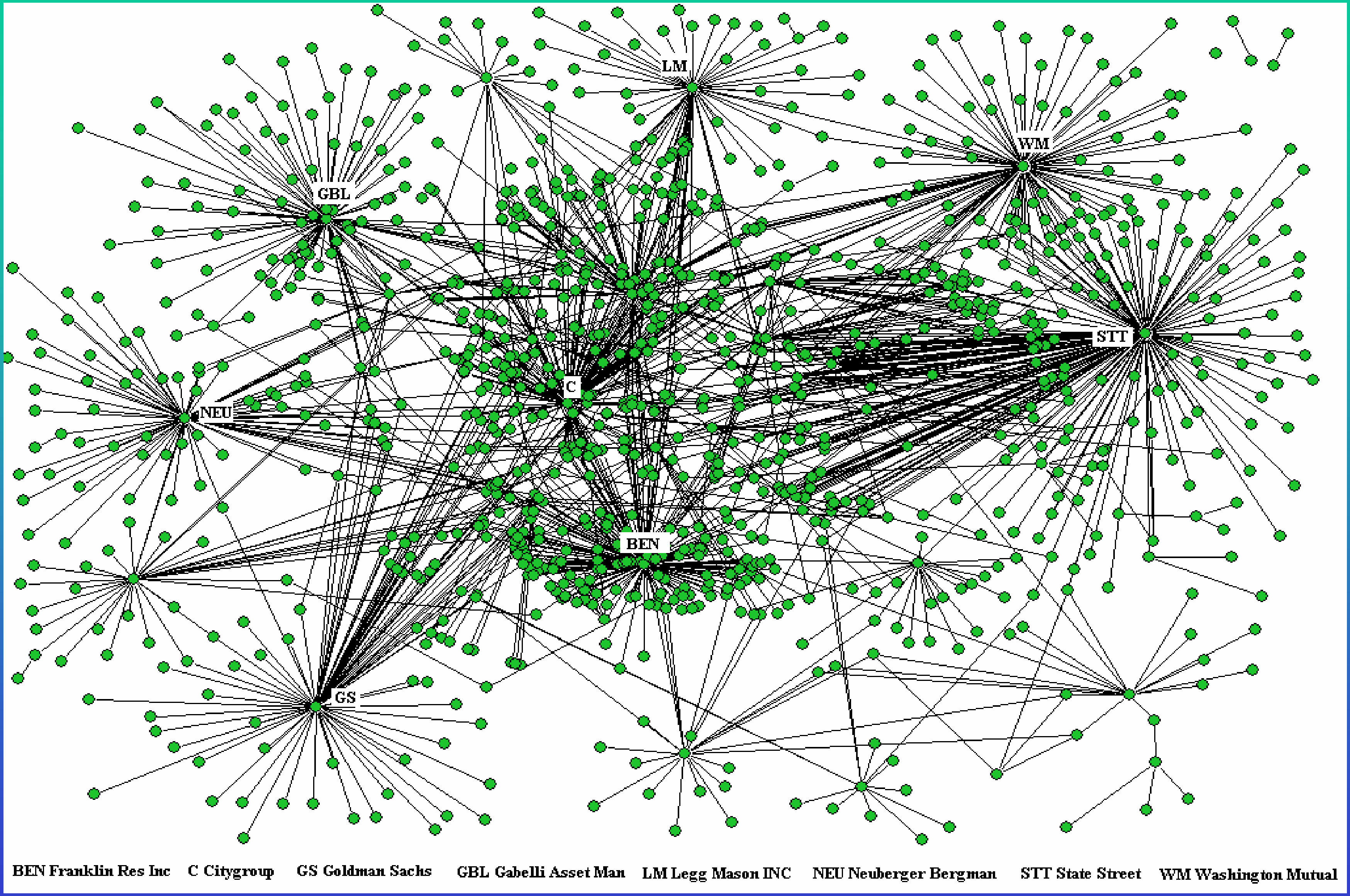
FITB Fifth Third Bank Corporate

FFBC First Financial Bank Coroporation

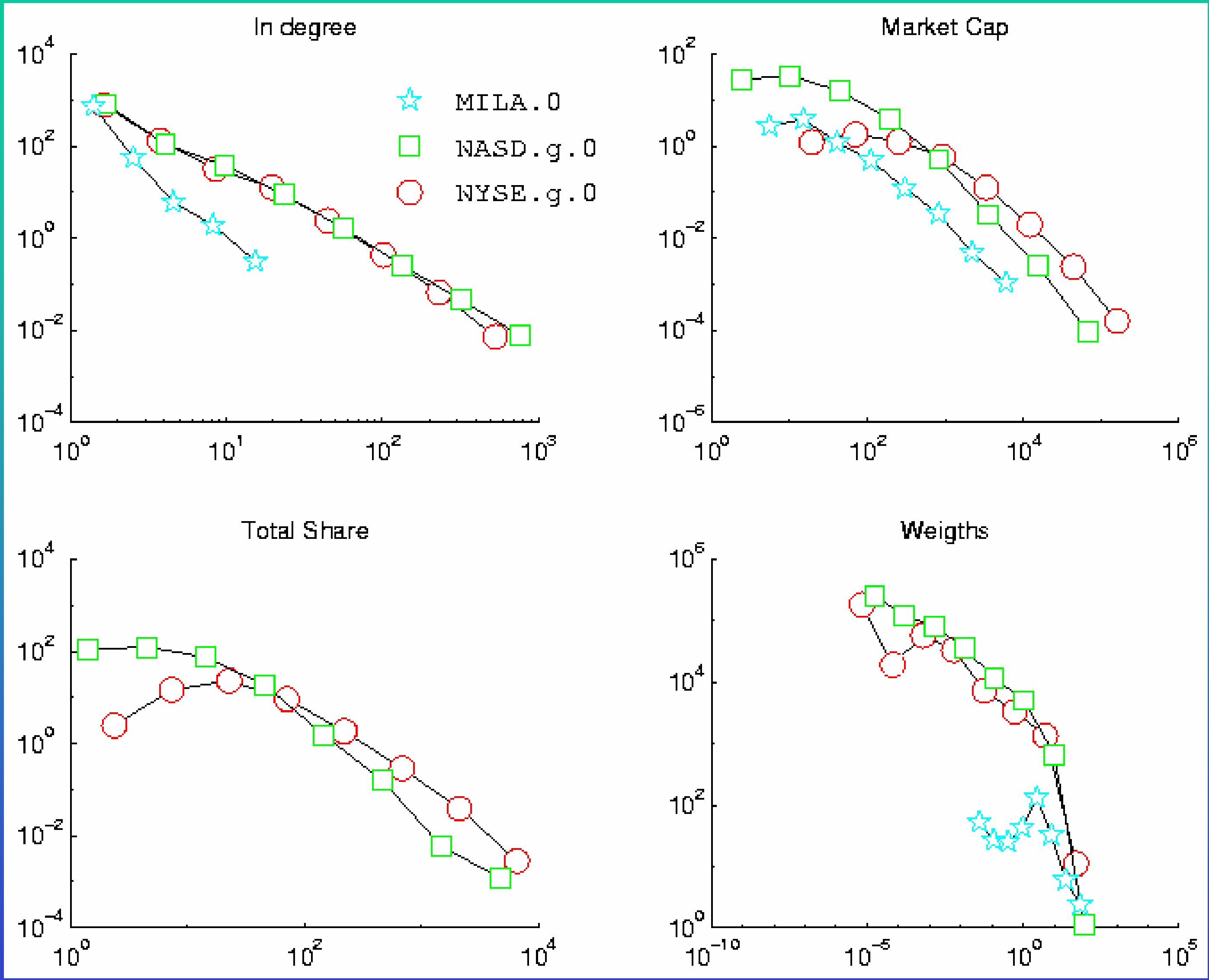
NTRS Northern Trust Corporation

SAFC Safeco Corporation

•2C Portfolio Composition



•2C Portfolio Composition



•2C Portfolio Composition

It is not only the topology that matters.

In this case as in many other graphs the weight of the link is crucial

$$SI \propto \frac{\sum_j w_{ij}^2}{\left(\sum_j w_{ij}\right)^2}$$

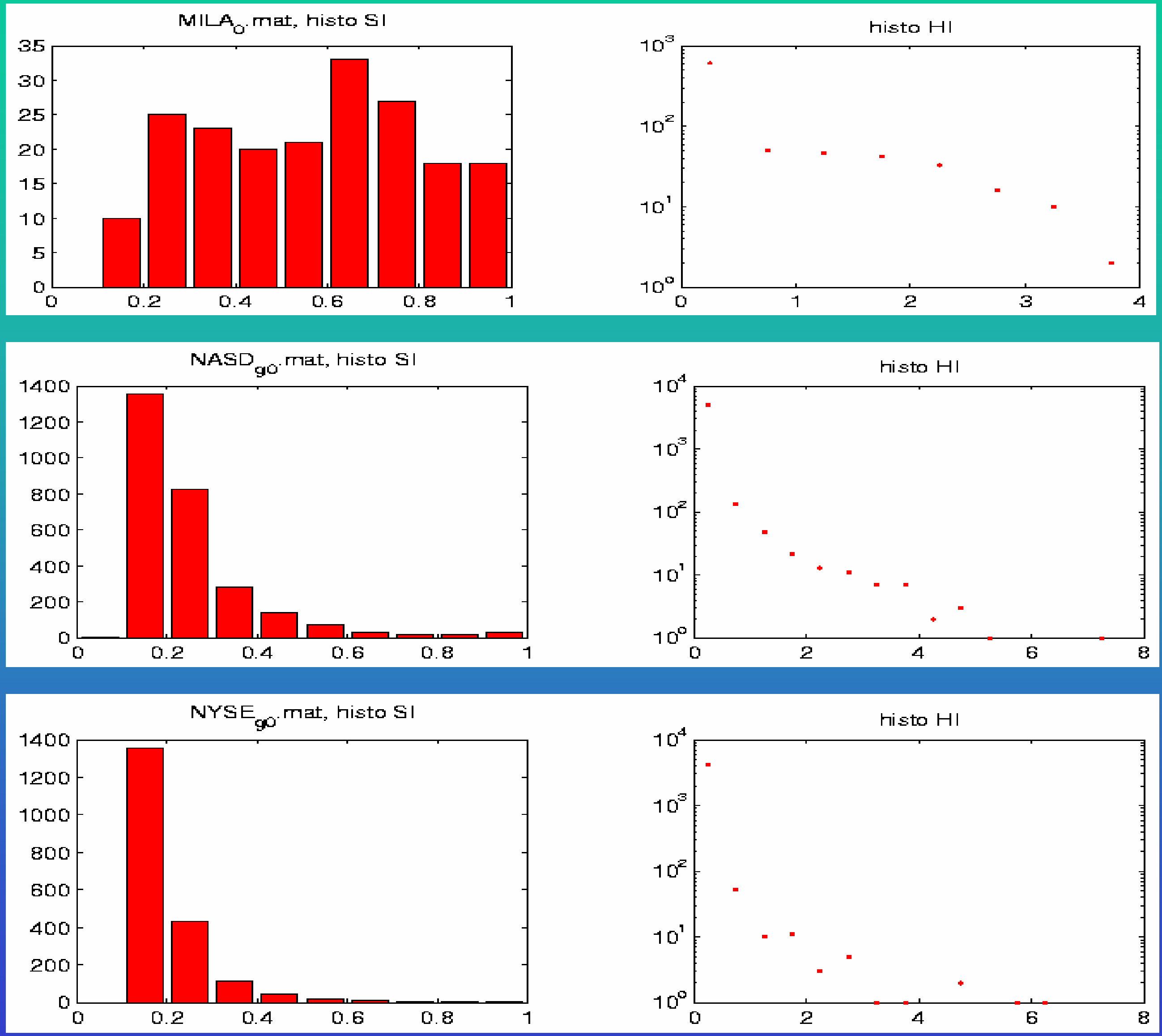
For every stock *i* you compute this quantity.
The sum runs over the different holders

- **If there is one dominating holder SI tends to one**
- **If all the holders have a similar part SI tends to 1/N**

$$HI(j) \propto \sum_i \frac{w_{ij}^2}{\left(\sum_l w_{il}\right)^2}$$

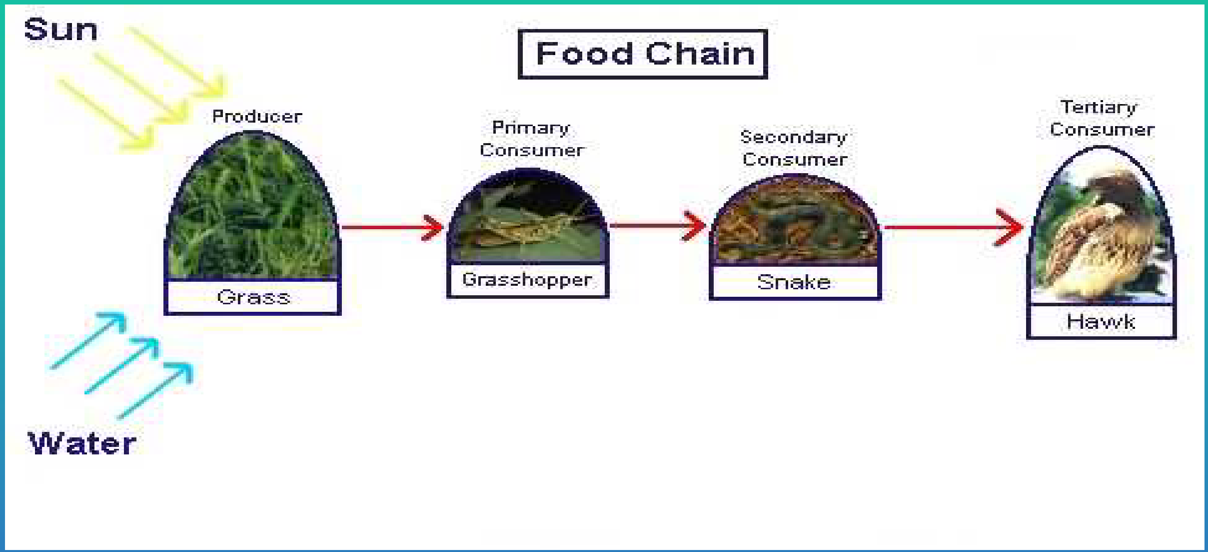
For every guy *j* you compute this quantity.
The sum at the denominator runs over the different holders of *i*
Then you sum on the different stocks in the portfolio
This gives a measure of the number of stocks controlled

•2C Stock correlation



•2D Food Webs

FOOD CHAIN = sequence of *predation relations* among different living species sharing the same physical space (Elton, 1927):



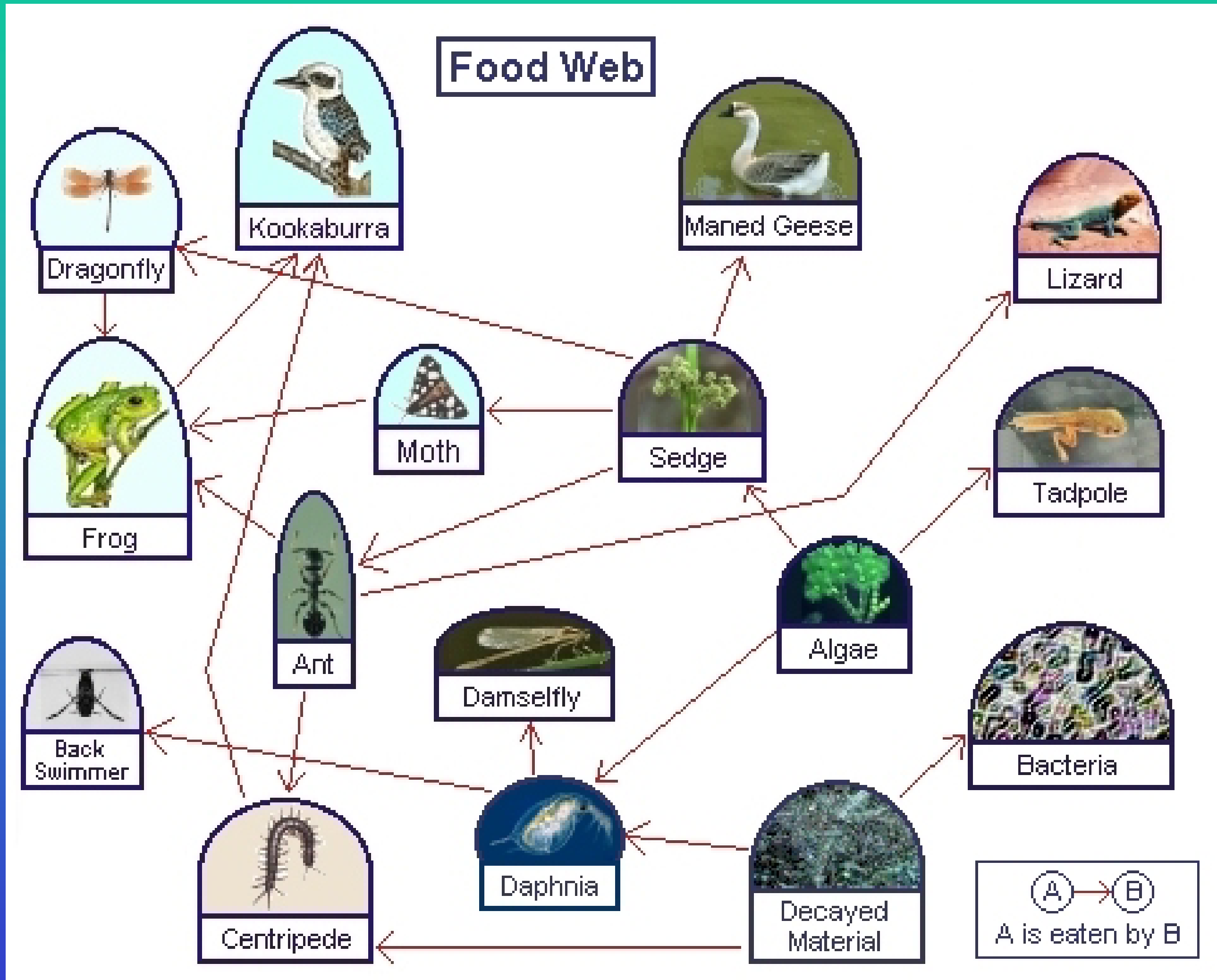
→ **Flow of matter and energy from prey to predator, in more and more complex forms;**

→ **The species ultimately feed on the abiotic environment (light, water, chemicals);**

→ **At each predation, almost 10% of the resources are transferred from prey to predator.**

•2D Food Webs

A series of different interconnected food chains form a food web



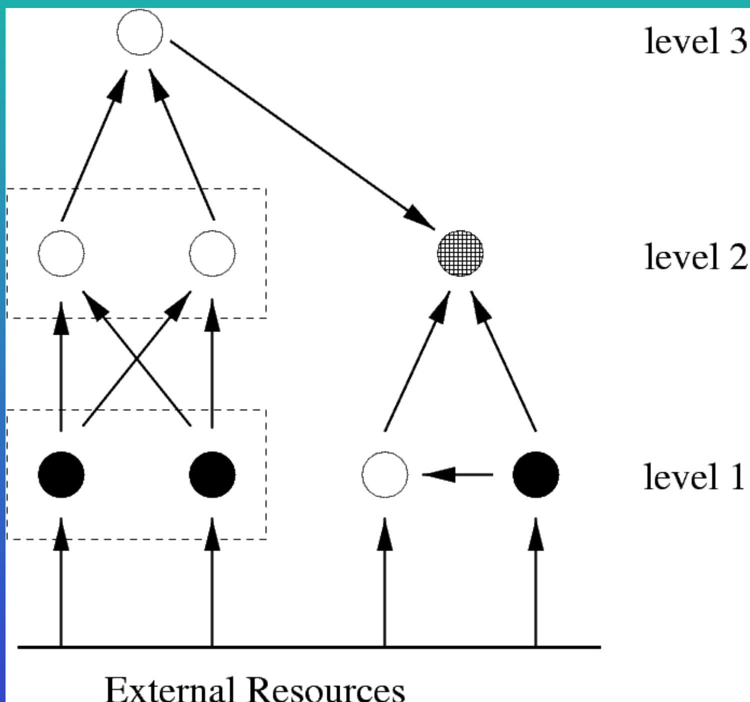
•2D Food Webs

Trophic Species:

Set of species sharing the same set of preys and the same set of predators (*food web* → *aggregated food web*).

Trophic Level of a species:

Minimum number of predations separating it from the environment.



Basal Species:

Species with no prey (B)

Top Species:

Species with no predators (T)

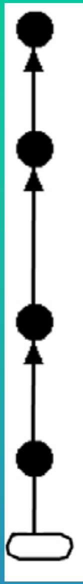
Intermediate Species:

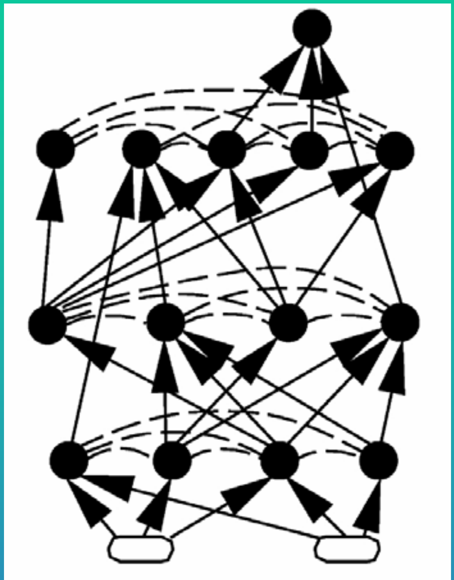
Species with both prey and predators (I)

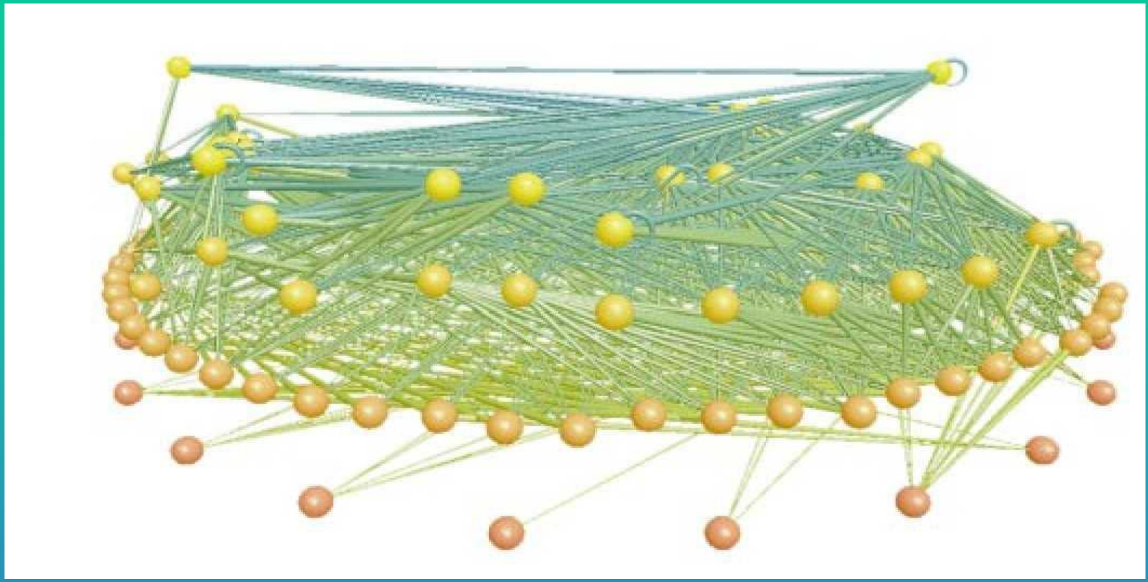
Prey/Predator Ratio = $\dfrac{B+I}{I+T}$

•2D Food Webs

* See Neo Martinez Group at http://userwww.sfsu.edu/~webhead/lrl.html







↓

Pamlico Estuary (North Carolina):
14 species

↓

Aggregated Food Web of Little Rock Lake (Wisconsin)*:
182 species → 93 trophic species

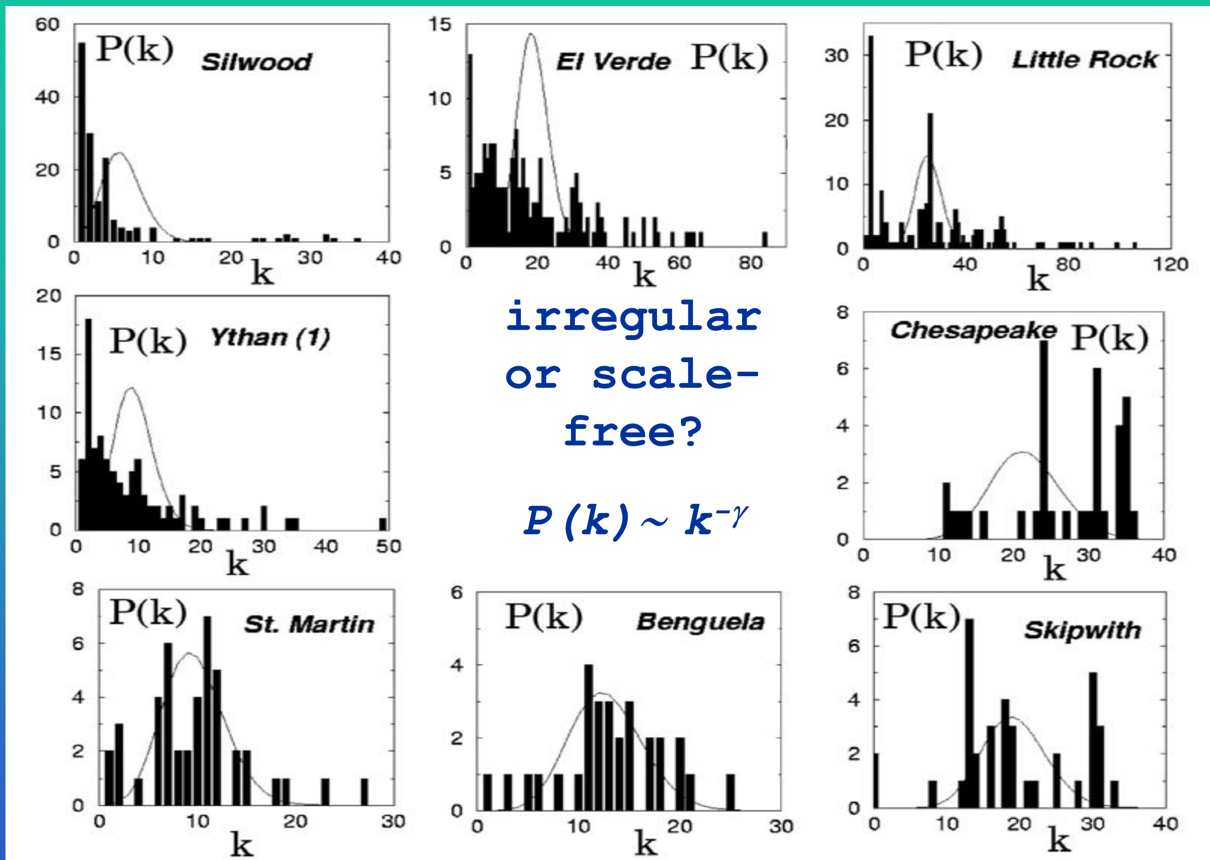
How to characterize the topology of Food Webs?

⇩

Graph Theory

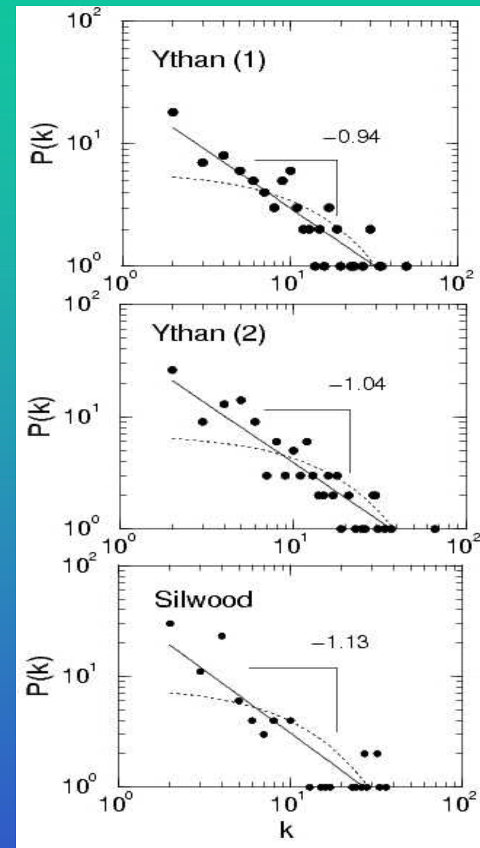
•2D Food Webs: Degree Distribution

Unaggregated versions of real webs:



irregular or scale-free?

$P(k) \sim k^{-\gamma}$

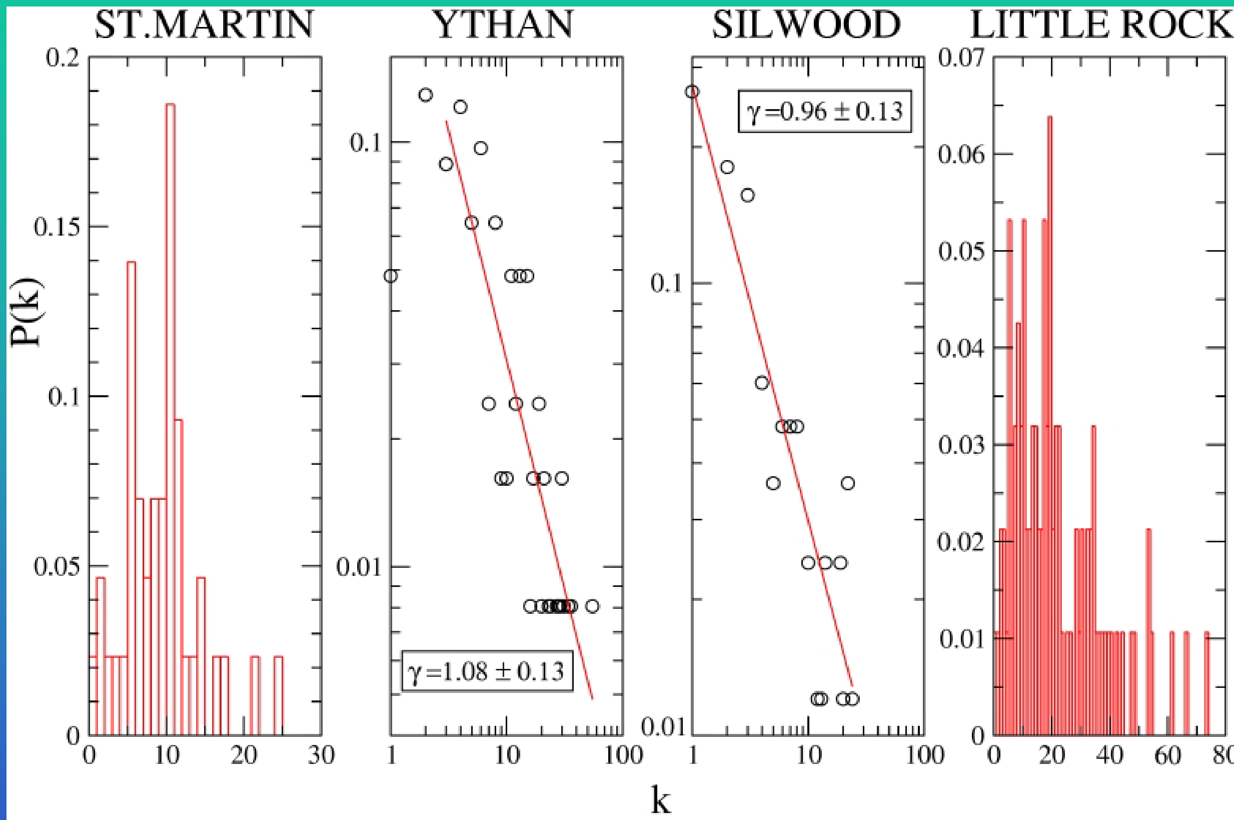


R.V. Solé, J.M. Montoya *Proc. Royal Society Series B* **268** 2039 (2001)

J.M. Montoya, R.V. Solé, *Journal of Theor. Biology* **214** 405 (2002)

•2D Food Webs: Degree Distribution

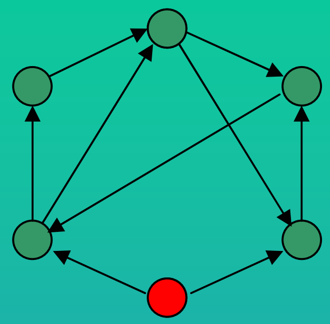
Aggregated versions of real webs:



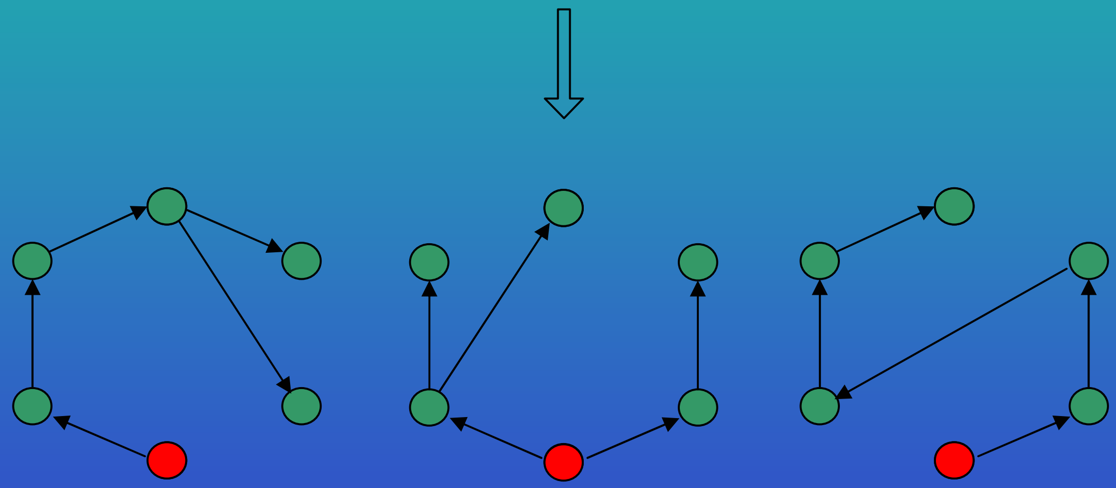
Same qualitative behaviour of their unaggregated counterparts.

We look for other quantities!.

•2D Food Webs: Spanning Trees of a Directed Graph



A *spanning tree* of a connected directed graph is any of its connected directed subtrees with the same number of vertices.



In general, the same graph can have more spanning trees with different topologies.

•2D Food Webs Spanning Trees from data

St.Martin's Island (Antilles):

44 species → 42 trophic species
224 links → 211 trophic links
(low taxonomic resolution)

Ythan Estuary (Scotland):

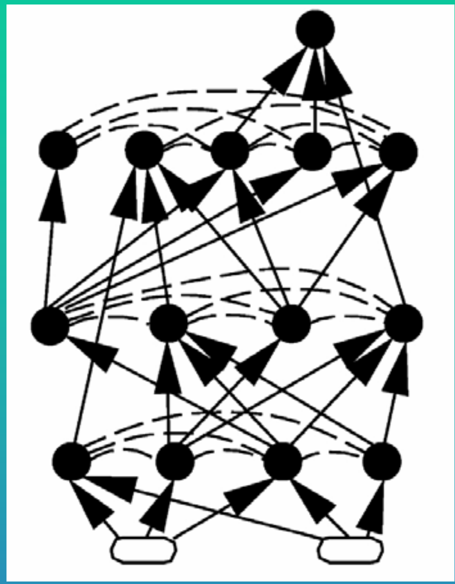
134 species → 123 trophic species
597 links → 576 trophic links
(taxonomic resolution : 88%)

Silwood Park (United Kingdom):

154 species → 83 trophic species
365 links → 215 trophic links
(taxonomic resolution : 100%)

Little Rock Lake (Wisconsin):

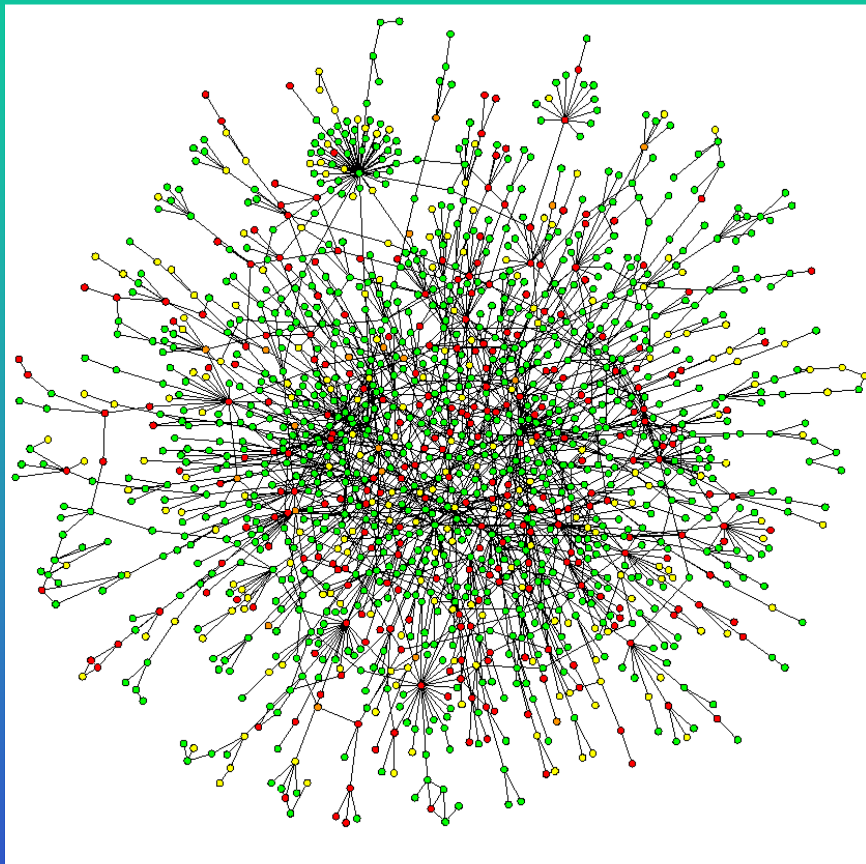
182 species → 93 trophic species
2494 links → 1046 trophic links
(taxonomic resolution : 31%)

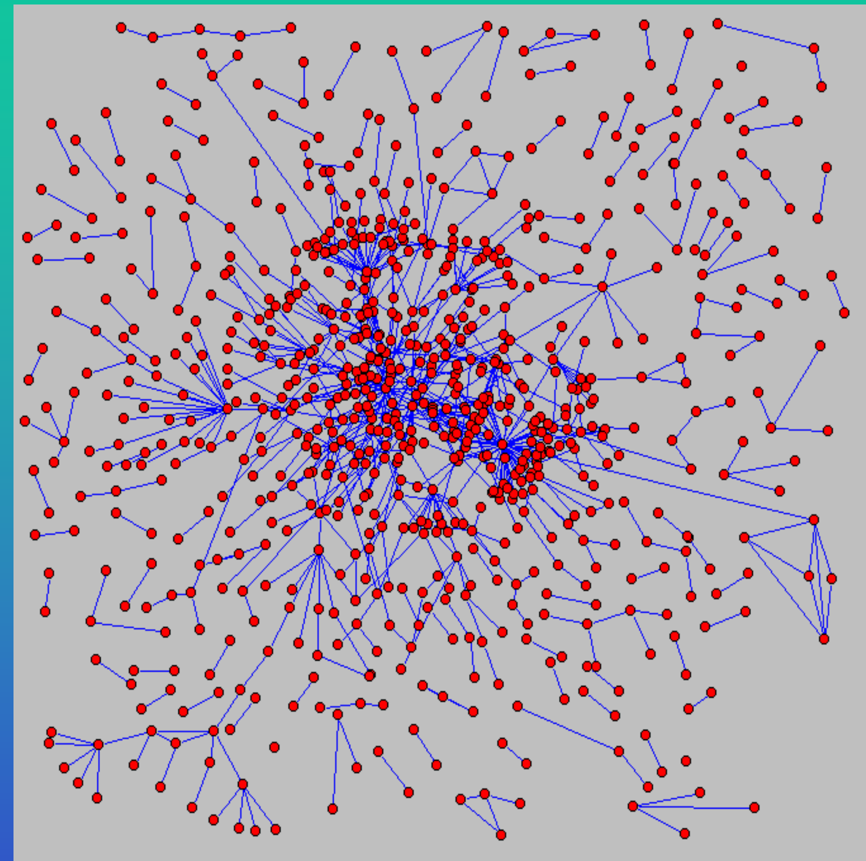


Spanning Tree:

All edges directed from level l_1 to levels $l_2 \leq l_1$ are removed

•2D Protein Interactions





Network of Interaction for the protein of Baker's Yeast (*Saccharomyces Cerevisiae)*

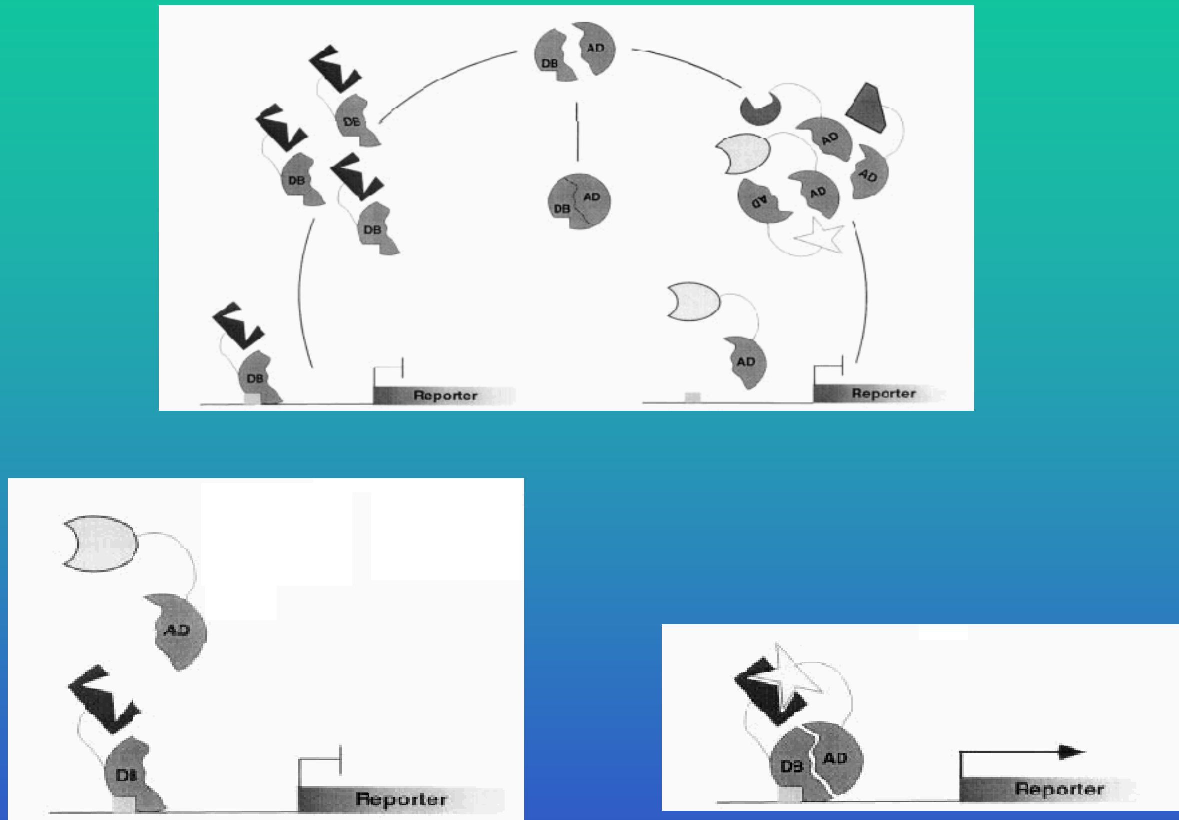
•2D Origin of Protein Networks

How do growth and preferential attachment apply to protein networks?

- **Growth**: genes (that encode proteins) can be, sometimes, duplicated; mutations change some of the interactions with respect to the parent protein
- **Preferential attachment**: the probability that a protein acquires a new connection is related to the probability that one of its neighbors is duplicated; proportional to its connectivity

A. Vazquez et al., *ComPlexUs* 1, 38-44 (2003)

•2D Two-hybrid method

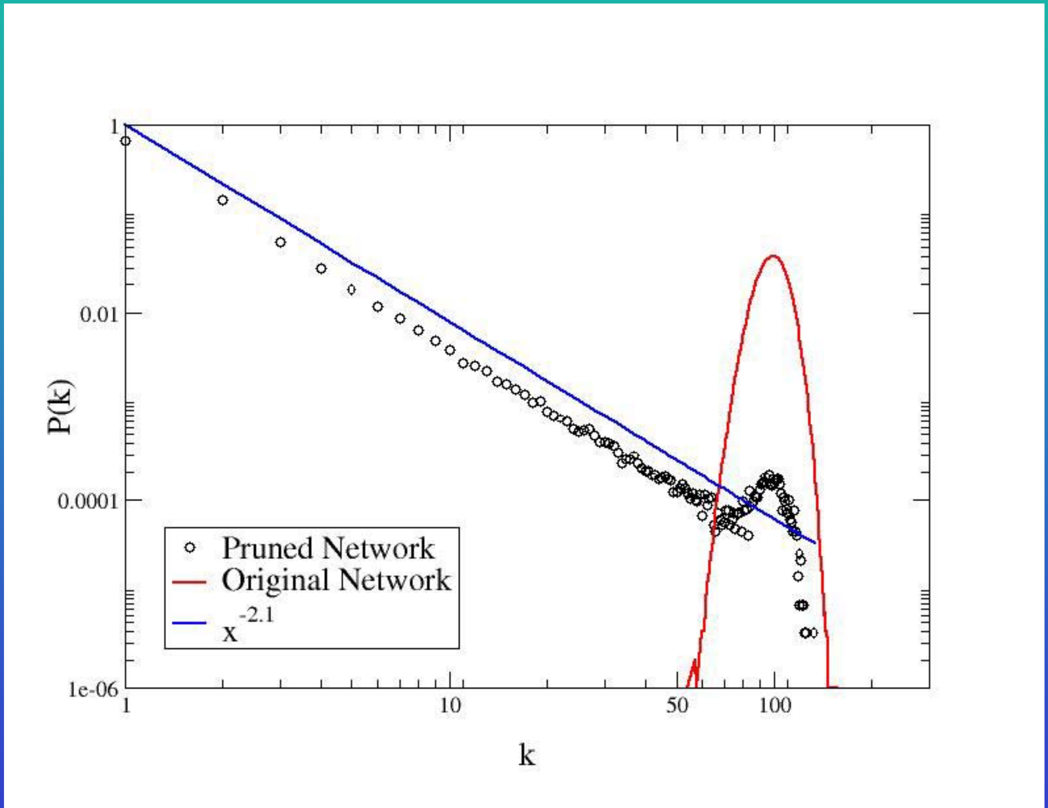


The two hybrid method way of detecting protein interactions

•2D More refined Models

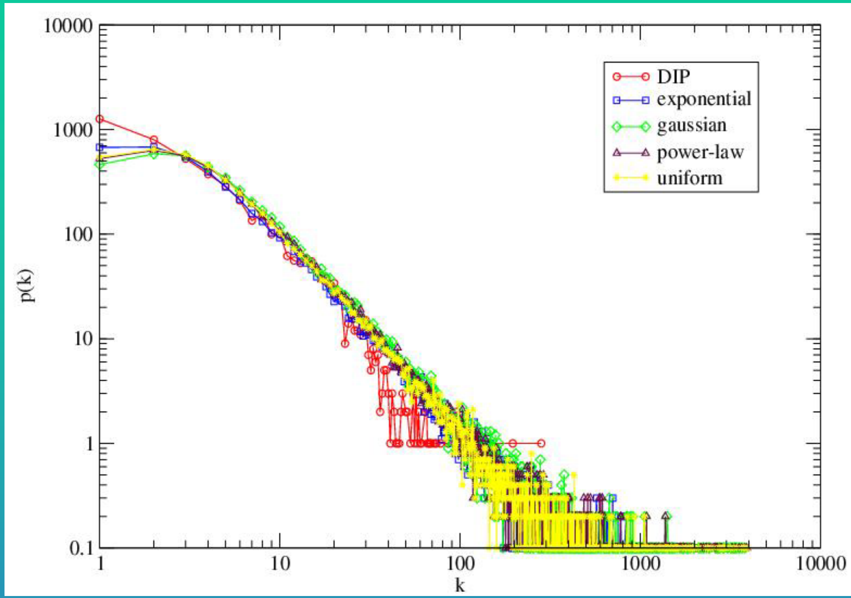
With the solvation free energies taken from an exponential probability distribution $p(f) = e^{-f}$, we obtain

$$P(k) \sim k^{-2}$$



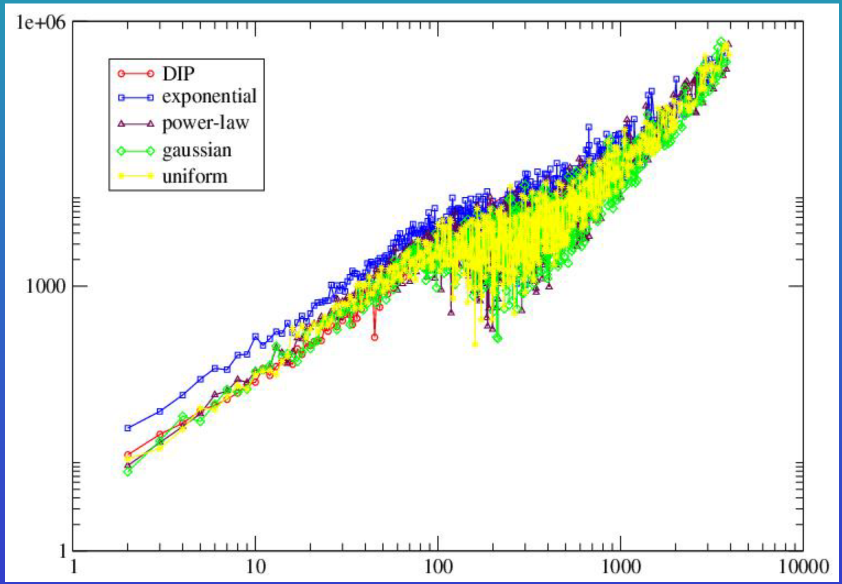
- The real network is random
- The detection method sees only pairs with large enough binding constants
- The binding constant is related to the solubilities of the two proteins
- Solubilities are given according to some distribution

•2D Protein Interactions

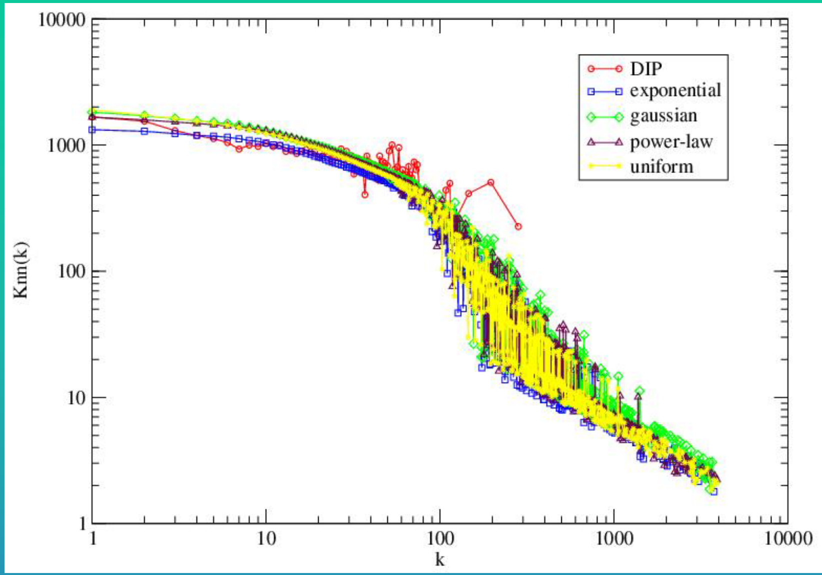


← **Scale-Free Degree distribution**

Scale-Free Betweenness b(k) →



•2D Protein Interactions



← neighbors degree per degree Knn(k)

Clustering per degree c(k) →